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Coregular Total Domination in Line Graph

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Abstract

In this paper we establish some results on Core gular total domination in line graph L(G). Let a dominating set D of V[L(G)] be a total dominating set of L(G). If the induced subgraph $< V[L(G)] - D >$ *is regular, then D is a Coregular total dominating set of L(G). The minimum cardinality of a minimal Coregular total dominating set is a Coregulat total domination number and denoted by* $\gamma_{crt}[L(G)]$.

Keywords: *Line graph, total dominating set, Coregular, Total dominating set, Coregular total, Domination number.*

1. Introduction

The graphs considered here are finite and simple. Any undefined term in this paper may be found in Harary [6]. We begin by recalling some standard definitions from domination theory. A subset D⊆ $V(G)$ is a dominating set of G, if every vertex of $V(G) - D$ has a neighbour in D. The domination number of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set.

For a comprehensive survey of dominations in graphs, see[7] . Restrained domination in graphs was introduced by Domke.al[5]. A dominating set S⊆ $V(G)$ is a restrained dominating set of a graph G if every vertex in V(G)-S has a neighbour in S as well as neighbour in V(G)-S. The restrained domination number γ_r(G) of G is the minimum cardinality of a restrained dominating set of G.

A dominating set S of a graph G is said to be a cototal dominating set of G if the induced subgraph $\langle V(G) - S \rangle$ has no isolated vertices. The cototal domination number of G , is denoted γ_{cot}(G) is the minimum cardinality of a cototal dominating set of G .Independently Kulli.et.al [8] initiate the study of cototal domination in graphs.

A subset D⊆ $V(G)$ is a double dominating set of G if every vertex of G is dominated by atleasttwo vertices of of D . The double domination number of G ,denoted by γ_{dd}(G) , is the minimum cardinality of adouble dominating set of G. The study of double domination in graphs was initiated by Harary and Haynes.[6]

A set S⊆ $V(G)$ is an independent dominating set of G, if every vertex in induced subgraph $\langle S \rangle$ is independent. The independent domination number i(G) of G is the the minimum cardinality of an independent dominating sets of G.See [1]

A dominating set S of a graph G is a total dominating set if the induced subgraph $\langle S \rangle$ contains no solitary edges. The total domination number $\gamma_t(G)$ is the least cardinality of atotal dominating set of G . See [3].

A set $S \subseteq E(G)$ is an edge dominating set of G, if every edge in E(G)-S is adjacent to atleast one edge in S. An edge dominating set S is connected if the subgraph induced by S is connected .The minimum cardinality of connected dominating set S is called the connected edge dominating number $\gamma_c^{|}$ of G.

A dominating set S of a graph G is perfect dominating set if every vertex of V-S is adjacent to exactly one vertex of S . The minimum cardinality of a perfect dominating set of G is a perfect domination number and is denoted by , $\gamma_p(G)$ see [4].

2. Results

we give the Coregular total domination number of line graphs of some standard graphs,which are straight forward in the following theorem.

Theorem 1:

1. For any Path P_p , with $p \geq 4$ vertices,

$\gamma_{crt}[L(P_{3n})] = 2n - 1$ where n = 1,2,3,…

$\gamma_{crt}[L(P_{3n+1})] = 2n$

$\gamma_{crt}[L(P_{3n+2})] = 2n$

2. For any Wheel W_p , with $p \geq 4$ vertices,

$\gamma_{crt}[L(W_p)] = p - 1$.

Theorem 2 : For any connected(p,q)graph G with $p \geq 3$ vertices ,

$$\gamma_{crt}L(G) \leq \left\lceil \frac{P}{2} \right\rceil + 2 \text{ , } G \neq W_p \text{ with } p \geq 8$$

Proof: Suppose $G = W_P$ with $p \geq 8$

$\gamma_{crt}L(G) > \left\lceil \frac{P}{2} \right\rceil + 2$ Hence $G \neq W_p$ with $p \geq 8$

Let V= $\{v_1, v_2, \ldots\ldots, v_n\}$be the vertex set of G with $|V|$ = P

Let D_1= $\{v_1, v_2, \ldots\ldots, v_m\}$is a dominating set of L(G).Suppose M = V[L(G)] $-D_1$

Further $D_2 \subset M$ and $\langle D_1 \cup D_2 \rangle$ has no isolates such that $N[D_1 \cup D_2]$ = V[L(G)].Then $D_1 \cup D_2$ forms a total dominating set of L(G).

Now assume D_3= [V[L(G)]$-\{D_1 \cup D_2\}$] and the induced subgraph $\langle D_3 \rangle$ is regular .Then D_3 is a γ_{crt} of L(G). Clearly it follows that

$|D_3| \leq \left\lceil \frac{P}{2} \right\rceil + 2$ and Hence $\gamma_{crt}L(G) \leq \left\lceil \frac{P}{2} \right\rceil + 2$.

Theorem 3 : For any connected(p,q)graph G with $p \geq 3$ vertices ,

$\gamma_{crt}L(G) + N \geq \gamma(G) + \gamma_t(G)$, Where N is the set of endedges , $G \neq P_p$and $G \neq C_p$.

Proof: Suppose $G = P_p$and $G = C_p$ .Then $\gamma_{crt}L(G) + N \ngeq \gamma(G) + \gamma_t(G)$.

Let F= $\{e_1, e_2, \ldots\ldots, e_m\} \subseteq E(G)$ be the set of all end edges in G with $|F|$ = N.

Let S_1= $\{v_1, v_2, \ldots\ldots, v_k\} \subseteq V(G)$ be the set of all end vertices in G .Suppose S_2= $\{v_1, v_2, \ldots\ldots, v_l\} \subseteq S_1$ be the minimum set of vertices , which covers all vertices in G,such that $N[S_2]$ = V(G). Then S_2 forms a dominating set of G. Further if the induced subgraph $\langle S_2 \rangle$ has no isolates.Then S_2 itself is a γ_t set of G. Suppose if $\deg(v_1) < 1$,then attach the vertices $w_i \in N(v_l)$ to make $\deg(v_1) \geq 1$ such that $\langle S_2 \cup \{w_i\} \rangle$ has no isolated vertex . Clearly $S_2 \cup \{w_i\}$ forms a total dominating set of G.

Let E_1= $\{e_1, e_2, \ldots\ldots, e_k\} \subseteq E(G)$, E_2= $\{e_1, e_2, \ldots\ldots, e_l\} \subseteq E(G)$. Then $\forall e_i \in E_1$ are incident with$\forall v_i \in S_2$,and $\forall e_j \in E_2$ are incident with $\forall v_j \in [S_2 \cup \{w_i\}]$.Let E_3= $\{e_1, e_2, \ldots\ldots, e_n\} = E(G)$. Then $\{u_1, u_2, \ldots\ldots, u_n\}$= V[L(G)] corresponding to the elements of E_3.Also $H_1 = \{u_1, u_2, \ldots\ldots, u_k\} \subset V[L(G)]$ corresponding to the elements of E_1 and $H_2 = \{u_1, u_2, \ldots\ldots, u_l\} \subset V[L(G)]$ corresponding to the elements of E_2. Suppose K$\} \subset V[L(G)]$ be the set of vertices which covers all the vertices of L(G) such that N[K] =

V[L(G)].Clearly K forms a dominating set of L(G).Suppose the induced subgraph $\langle K\rangle$ has no isolated vertices then K forms a total dominating set of L(G).

If the induced subgraph $\langle V[L(G)]-K\rangle$is regular ,then K is a γ_{crt} set of L(G).

If $|H_1|<|K|$ and $|H_2|<|K|$ then it is easily show that $|H_1\cup H_2|<|K|+|F|$ Which gives $\gamma_{crt}L(G)+\mathrm{N}\geq\gamma(G)+\gamma_t(G)$.

Next theorem relates $\gamma_{crt}L(G)$ in terms of dominating set of L(G) and edges of G.

Theorem 4 : For any connected(p,q)graph G with $p\geq 3$ vertices ,

$\gamma_{crt}L(G)+\gamma[L(G)]\leq$ q , G$\neq C_p$ with p≥ 7

Proof: If G$=C_p$ with p≥ 7 vertices , $\gamma_{crt}L(G)+\gamma[L(G)]>$ q

Hence G$\neq C_p$ with p≥ 7

Let E= $\{e_1,e_2,\ldots\ldots,e_n\}$ be the edge set of G with $|E|$ = q

Let H be the set of vertices with deg$(u_i)\geq 2$, $\forall u_i\in H$, 1$\leq i\leq n$ in L(G).Further let $H_1=\{u_1,u_2,\ldots\ldots,u_k\}\subseteq V[L(G)]$ such that dist$(u,v)\geq 2$.Then there exists a minimal set of vertices H_2 in L(G),such that $\forall u_i\in V[L(G)]-H_2$ is adjacent to atleast one vertex of H_2. Hence H_2 is minimal γ set of L(G).Suppose the induced subgraph $\langle H_2\rangle$ has no isolated vertices. Then H_2 itself is a minimal total dominating set of L(G). If in the induced subgraph $\langle V[L(G)]-H_2\rangle$ every vertex has same degree , then H_2 is a γ_{crt}- set of L(G).If not add the set of vertices $\{w_i\}\in V[L(G)]-H_2$such that the induced subgraph $\langle V[L(G)]-H_2\cup\{w_i\}\rangle$ is regular. Hence $|H_2\cup\{w_i\}|+|H_2|\leq|E|$ which gives

$\gamma_{crt}L(G)+\gamma[L(G)]\leq$ q

The following theorem gives the relation of our concept with the edge domination number and restrained domination number of a graph G.

Theorem 5 : For any connected(p,q)graph G with $p\geq 3$ vertices ,

$\gamma_{crt}L(G)\leq\gamma'(G)+\gamma_r(G)$, G$\neq W_p$ with p ≥ 5

Proof: If G$=W_p$ with p ≥ 5, then $\gamma'(G)+\gamma_r(G)<\gamma_{crt}L(G)$.

Let E_1= $\{e_1,e_2,\ldots\ldots,e_k\}\subseteq$ E(G) such that for each $e_i\in E_1$, i = 1,2,3….k, N$(e_i)\cap E_1=\emptyset$. Then $|E_1|=\gamma'(G)$.

Let A = $\{v_1,v_2,\ldots\ldots,v_m\}$ be the set of endvertices in G, and B$\subseteq V(G)-A$.Then there exists a vertex set H$\subseteq$B such that $\forall u_k\in[V(G)-H\cup A]$ is adjacent to atleast one vertex of { $H\cup A$ }adjacent to atleast one vertex of { $V(G)-H\cup A$}.Then $\{H\cup A\}$ is a γ_r set of G. Suppose M$\subset E_1$ and $M_1\subset E(G)-E_1$, then in L(G) , {M}$\cup\{M_1\}\subset V[L(G)]$.

Now assume $\forall u_i\in\{V[L(G)]-\{\mathrm{M}\cup M_1\}]\}$is adjacent to atleast one vertex of $\{\mathrm{M}\cup M_1\}$ and N[M $\cup M_1$] = V[L(G)].The $\{\mathrm{M}\cup M_1\}$ is a minimal γ_t-set of L(G).

Suppose the induced subgraph $\langle V[L(G)]-\{\mathrm{M}\cup M_1\}\rangle$ is regular.Then $\{\mathrm{M}\cup M_1\}$ is a γ_{crt}set of L(G).

Otherwise add the set of vertices$\{u_k\}$from $V[L(G)]-\{\mathrm{M}\cup M_1\}$ which makes the induced subgraph $\langle V[L(G)]-\{\mathrm{M}\cup M_1\}\cup\{u_k\}\rangle$ regular, Hence

$|\{\mathrm{M}\cup M_1\}\cup\{u_k\}|\leq|E_1|+|H\cup A|$ which gives

$\gamma_{crt}L(G)\leq\gamma'(G)+\gamma_r(G)$.

Within the ensuring the theorem we relates our concept to $\gamma_p(G)$ and vertices of graph G.

Theorem 6 : For any connected(p,q)graph G with $p\geq 3$ vertices ,

$\gamma_{crt}L(G))+\gamma_p(G)\leq$ p , G$\neq C_p$ where p$=2n+1$, n= 2,3,4……

Proof: Suppose **i**f G $=C_p$ with p = 2n+1, n=2,3,4…..,then $\gamma_{crt}L(G)+\gamma_p(G)\leq$ P+1 a contradiction to the above result .

Let V = $\{v_1,v_2,\ldots\ldots,v_n\}$ be the vertex set of G with $|V|=p$

Let V_1 = $\{v_1,v_2,\ldots\ldots,v_r\}\subseteq$V(G) such that N$(v_i)\cap$ N(v_j) = $\emptyset$, $\forall i,j\in V_1$. Then V_1 is minimal perfect dominating set of graph G. Let A = $\{v_1',\ v_{21}',\ldots\ldots v_n'\}\subseteq V[L(G)]$ be minimal dominating set of L(G). Suppose there exists a minimal set B= $\{v_1',\ v_{21}',\ldots\ldots v_m'\}\in N(A)$ such that the induced subgraph $\langle A\cup B\rangle$ has no isolated vertex. Further if $A\cup B$

covers all vertices in L(G), then $A \cup B$ forms a minimal total dominating set of L(G). If the induced subgraph $\langle V[L(G)] - \{A \cup B\}\rangle$ is regular , then $\{A \cup B\}$ itself is a γ_{crt} of L(G).Hence $|A \cup B| + |V_1| \leq |V|$ which gives $\gamma_{crt}L(G)) + \gamma_p(G) \leq$ p .

Next theorem establish a relation between $\gamma_{crt}L(G)$ and independent domination set of graph G with our concept.

Theorem 7 : For any connected(p, q)graph G with $p \geq 3$ vertices ,

$$\gamma_{crt}L(G)) + i(G) \leq \gamma_{crt}(G) + \beta_0(G) .$$

Proof: Let D = $\{v_1, v_2, \ldots.., v_k\} \subseteq$V(G)be a minimal dominating set of G. If $\forall v_i \in$ D, deg(v_i)= 0 ,then D is a independent dominating set of G.

Suppose L = $\{v_1, v_2, \ldots.., v_m\} \subseteq$ V(G) be the minimum set of vertices with dist$(u, v) \geq 2$ and N(u)∩ $N(v) \neq x$, $\forall u, v \in$ L and $x \in V(G) - L$. Clearly $|L| = \beta_0(G)$. Let F= $\{v_1, v_2, \ldots.., v_k\} \subset$ V(G) be the set of all endvertices in G and $V' = V - F$.

Suppose $D^1 \subseteq V^1$ be a minimal dominating set of G. Further if for some $\{v_i\} \in$ N(D') and $\langle D' \cup \{v_i\}\rangle$ forms a minimal total dominating set of G. If $\langle V - D' \cup \{v_i\}\rangle$ is regular then $D' \cup \{v_i\}$ is a γ_{crt}-set of G.

Further let A be a vertex set of L(G).Now in L(G),let $A_1 = \{u_1, u_2, \ldots.., u_k\} \subset V[L(G)]$ be the set of vertices corresponding to edges which are incident to the vertices of L in G. Also, the set $B = \{u_1, u_2, \ldots.., u_m\} \subset V[L(G)]$ be the set of vertices corresponding to edges which are incident to the vertices in V_1 in G. Suppose $A_1 \subseteq A$, such that $\forall u_i \in A_1$ is adjacent to atleast one vertex of A_1 such that N[A_1]= V[L(G)].Then A_1 forms total dominating set of L(G).Suppose the induced subgraph $\langle V[L(G)] - A_1\rangle$ is regular ,then A_1 is a γ_{crt} -set of L(G).If not add the set of vertices $\{u_k\}$from $\{V[L(G)] - A_1\}$ which makes the induced subgraph $\langle V[L(G)] - A_1 \cup \{u_k\}\rangle$ regular. Hence

$|A_1 \cup \{u_k\}| + |D| \leq |D' \cup \{v_i\}| + |L|$ which gives $\gamma_{crt}L(G)) + i(G) \leq \gamma_{crt}(G) + \beta_0(G)$.

Theorem 8 : For any connected(p, q)graph G with $p \geq 3$ vertices ,

$$\gamma_{crt}L(G) \geq \gamma_c^1(G).$$

Proof: Let E_1 = $\{e_1, e_2, \ldots.., e_k\} \subseteq E$(G) which covers all sdges of G such that N[E_1]= E(G).Then $E_1$is a minimal edge dominating set of G. Suppose K= $\{v_1, v_2, \ldots.., v_n\}$be the set of all endvertices of G. Let E_2 = $\{e_1, e_2, \ldots.., e_m\}$ be the set of all edges which are not incident to the vertices of K and $\forall e_i \in E_2$ is adjacent to atleast one edge of E(G)$-E_2$. If the induced subgraph E_2 has exactly one component ,then E_2forms a connected edge dominating set of G.

Now in L(G), let D = $\{u_1, u_2, \ldots.., u_m\} \subseteq V[L(G)]$ be the set $\{u_j\} \leftrightarrow \{e_j\} \in E_1(G)$, suppose D be the minimal set of vertices with N[D] =V[L(G)] and $\forall u_m \in D$.Then D is a γ-set of L(G).

Now consider H⊂ $V[L(G)]$ where H∈N(D) and there exists $H_1 \subset H$ such that $\langle H_1 \cup D\rangle$has no isolates. Clearly $\{H_1 \cup D\}$ is a total dominating set of L(G).

In the induced subgraph, if $\forall v_i \in \langle V[L(G) - \{H_1 \cup D\}\rangle$ has same degree, then $\{H_1 \cup D\}$ is a γ_{crt}-set of L(G) . Hence $|H_1 \cup D| \geq |E_1|$ which forms

$$\gamma_{crt}L(G) \geq \gamma_c^1(G).$$

Theorem 9 : For any connected(p, q)graph G with $p \geq 3$ vertices with m endvertices ,

$$\gamma_{crt}L(G)) + \Delta'(G) \geq \left\lceil \frac{q+m}{2} \right\rceil - 1.$$

Proof: Let E = $\{e_1, e_2, \ldots.., e_m\}$be the edge set of G, with $|E| = q$. Suppose F= $\{v_1, v_2, \ldots.., v_m\} \subset V(G)$be the set of all endvertices in G with $|F| = m$. Now assume E_1 = $\{e_1, e_2, \ldots.., e_j\} \subseteq E$(G) be the set of all nonend edges in G,then there exists atleast one edge with maximum degree $\Delta'(G)$ in E_1.

Further let H = $\{u_1, u_2, \ldots\ldots, u_n\}$ be the vertex set of L(G) corresponding to the elements of E(G).Suppose H_1 = $\{u_1, u_2, \ldots\ldots, u_m\} \subseteq H$ be the minimum set of vertices which covers all the vertices in L(G). Suppose deg$(u_i) \geq 1$, $\forall u_i \in H_1$ then H_1 is a γ_t-set of L(G).Otherwise if deg$(u_i) < 1$,then attach the vertices $\forall u_j \in N(u_i)$ to make deg$(u_i) \geq 1$such that $\langle H_1 \cup \{u_j\}\rangle$ has no isolated vertex. Clearly $H_1 \cup \{u_j\}$ forms a total dominating set of L(G). Further if $\langle V[L(G)] - H_1 \cup \{u_j\rangle$ is regular,then $H_1 \cup \{u_j\}$is a Coregular total dominating set of L(G). Hence $\left|H_1 \cup \{u_j\right| + E_1 \geq \left\lceil\frac{|F|+|E|}{2}\right\rceil - 1$ which gives

$\gamma_{crt}L(G)) + \Delta'\ (G) \geq \left\lceil\frac{q+m}{2}\right\rceil - 1.$

Theorem 10 : For any connected(p,q)graph G with $p \geq 3$ vertices with m endvertices , $\gamma_{crt}L(G)) + \gamma_{cot}(G) \geq \gamma_{dd}(G)$.

Proof: Suppose P = $\{v_1, v_2, \ldots\ldots, v_k\} \subseteq V(G)$such that N[P] = V(G).Then P is a dominating set of G.If the induced subgraph $\langle V - P\rangle$has no isolates, then P is a γ_{cot}-set of G.

Now consider $P_1 = V(G) - P$ and P_2 = $\{v_1, v_2, \ldots\ldots, v_i\} \subseteq P_1$, then $D^d = P \cup P_2$ forms a double dominating set of G.

Now in L(G), let D = $\{u_1, u_2, \ldots\ldots, u_n\} \subseteq V[L(G)]$ be the set $\{u_j\} \leftrightarrow \{e_j\} \in E(G)$,
1≤ $j \leq n$ where $\{e_j\}$ are incident with the vertices of $\{P \cup P_2\}$ .Suppose D be a minimal set of vertices with N[D]= V[L(G)] and $\forall u_n \in D$. Then D is γ-set of L(G). Suppose the induced subgraph $\langle D\rangle$has no isolated vertices , then D is a γ_t-set of L(G). If the induced subgraph $\langle V[L(G)] - \{D\}\rangle$ is regular, then D itself is a γ_{crt}set of L(G).Hence $|D| + |P| \geq |P \cup P_2|$ which attains $\gamma_{crt}L(G)) + \gamma_{cot}(G) \geq \gamma_{dd}(G)$.

Theorem 11 : For any connected(p,q)graph G with $p \geq 3$ vertices, $\gamma_{crt}L(G)) + \alpha_0(G) \leq P + \delta(G)$. G≠ W_p , G≠ C_p

Proof: Suppose G $= W_p$ and G $= C_p$ .Then $\gamma_{crt}L(G)) + \alpha_0(G) \nleq P + \delta(G)$.

Let $V = \{v_1, v_2, \ldots\ldots, v_n\}$ be the vertex set of G with $|V|$ = p.

Suppose B = $\{v_1, v_2, \ldots\ldots, v_m\} \subseteq V(G)$, deg$(v_i)$ ≥ 2 ∀ $v_i \in B$ be the set of vertices which covers all the edges in G. Clearly B firms a vertex covering set of G. Let v be a vertex with minimum degree $\delta(G)$ in G. Further let D = $\{u_1, u_2, \ldots\ldots, u_n\} = V[L(G)]$.Let D_1 = $\{u_1, u_2, \ldots\ldots, u_p\} \subseteq V[L(G)]$ be the set of all nonend vertices in L(G). Suppose $D_2 \subseteq D_1$ be the minimum set of vertices which covers all the vertices in L(G).If the induced subgraph $\langle D_2\rangle$ has no isolates.Then D_2 forms a total dominating set of L(G).Otherwise,if deg$(u_j) < 1, \forall\ u_j \in D_2$ attach the vertices $w_i \in N(u_j)$ to make deg$(u_j) \geq 1$, such that the induced subgraph $\langle D_2 \cup \{w_i\}\rangle$ has no isolates. Clearly $\langle D_2 \cup \{w_i\}\rangle$forms a minimal total dominating set of L(G).If the induced subgraph$\langle V[L(G)] - D_2 \cup \{w_i\}\rangle$ is regular then $D_2 \cup \{w_i\}$ is a γ_{crt} set of L(G).Hence $|D_2 \cup \{w_i\}| + |B^{|}| \leq |V|\ + \delta(G)$.
which gives the result $\gamma_{crt}L(G)) + \alpha_0(G) \leq P + \delta(G)$.

A edge dominating set D of a line graph L(G) is a coregular edge dominating set if the induced subgraph $|E[L(G)] - D|$is regular.The coregular edge domination number is the minimum cardinality of a coregular edge dominating set of L(G) and is denoted by $\gamma_{cr}^{|}L(G)$. **Theorem 11 :** For any connected(p,q)graph G with $p \geq 3$ vertices $\gamma_{crt}L(G)) + \gamma^{|}(G) \geq \gamma_{cr}^{|}(G)$.

Proof: Let E = $\{e_1, e_2, \ldots\ldots, e_n\}$be the edge set of G, Now consider Let E_1 = $\{e_1, e_2, \ldots\ldots, e_m\} \subseteq E(G)$be the set of edges with maximum edge degree and E_2 = $\{e_1, e_2, \ldots\ldots, e_k\} \subseteq E(G)$be the set of edges with minimum edge degree.

Suppose $E_1^{|} \subseteq E_1$ and $E_2^{|} \subseteq E_2$ if every edge in $\{E_1^{|} \cup\ E_2^{|}\}$ is adjacent to an edge in E(G)$-\{E_1^{|} \cup\ E_2^{|}\}$ then $\{E_1^{|} \cup\ E_2^{|}\}$forms a $\gamma^{|}$set of G.

Now in L(G), let D = $\{u_1, u_2, \ldots\ldots, u_m\} \subseteq V[L(G)]$be the set of vertices corresponding to the edges which are incident to the vertices of $\{E_1^{|} \cup E_2^{|}\}$in G .Suppose $D_1 \subseteq D$ such that $\forall u_j \in D_1$is adjacent to atleast one vertex of $D_1$and deg(u_i)≥ 1 such that N[D_1]= V[L(G)].Then D_1forms a total dominating set of L(G).Suppose the induced subgraph $\langle V[L(G)] - D_1 \rangle$ is regular .Then $D_1$is γ_{crt}-set of L(G).If not attach the set of vertices $\{u_j\}$from $\{V[L(G)] - D_1\}$which makes the induced subgraph $\langle V[L(G)] - D_1 \cup \{u_j\} \rangle$ is regular.

Hence $|D_1 \cup \{u_k\}| + |\gamma^{|}(G)| \geq \left|E_1^{|} \cup E_2^{|}\right|$ .Which gives the result

$\gamma_{crt} L(G)) + \gamma^{|}(G) \geq \gamma_{cr}^{|}(G)$.

Lemma: For any cycle C_p with $p \geq 3$ vertices

$\gamma_{crt} L(C_p)) = \gamma_t^{|}(C_p) = \gamma_c(C_p)$.

4. Conclusion

In this work, we looked line graphs with Coregular Total Domination Number. These results show a significant correlation between the Coregular total domination number in a line graph and many factors, such as the Perfect domination number, edge domination number , connected domination number, the Restrained domination number and Double domination number, of a straightforward, undirected graphs with no loops. The idea behind Coregular , the regularity of the vertices of V[L(G)] − D. D is the total domination number of line graph L(G). Here, we have some broad conclusions on the ideal of Coregular total Domination number. Additionally, its relation with additional dominating parameters were discovered.

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