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Comparative Exploration of Tuberculosis Transmission in High-Risk and Low-Risk Populations by Enhanced Numerical Algorithm

Gunasekaran.M[1], Alamelu.K[2*]

[1]PG Department of Mathematics, Sri Subramaniyaswamy Government Arts College, Tiruttani- 631 209, India.

[2]Department of Mathematics, Nazareth College of Arts and Science, Avadi, Chennai–600062, India.

[2]PG Department of Mathematics, Sri Subramaniyaswamy Government Arts College, Tiruttani- 631209, India., alameluiyappan@gmail.com

***[*]Corresponding Author:** Gunasekaran.M*

*[*]PG Department of Mathematics, Sri Subramaniyaswamy Government Arts College, Tiruttani- 631209, India. *alameluiyappan@gmail.com*

Abstract

This study compares the transmission of tuberculosis (TB) in high and low-risk populations by employing the 6th Order Runge-Kutta algorithm and the Susceptible-Vaccinated-Exposed-Infected (SVEI) model. Mycobacterium tuberculosis, the bacterium causing TB, is a global health concern with varying incidence among different demographic groups. Our research focuses on two distinct populations: high-risk individuals, who have had greater exposure to TB, and low-risk persons with minimal contact. The SVEI model is used to depict the dynamics of TB transmission, considering variables such as vaccination and latency duration. In this study, we introduce a novel methodology that considers only the true positives in each SVEI model category. This approach involves accounting solely for those accurately classified as susceptible, immunized, exposed, or infected, utilizing the sixth-order Runge-Kutta method. By employing this method, we can forecast future trends in TB transmission in both populations. Preliminary findings reveal significant differences in the dynamics of TB transmission between high- and low-risk populations. These results could potentially impact public health strategies for TB control by emphasizing the necessity of targeted interventions that consider population-specific risk levels. However, further investigation is necessary to validate these conclusions.

Keywords: Tuberculosis, High-Risk Population, Low-Risk Population, SVEI Model, 6th Order Runge-Kutta Method

1. Introduction

Worldwide, tuberculosis poses a significant challenge [15]. However, the burden of tuberculosis disease disproportionately affects impoverished nations. In addition to the six nations mentioned previously, several countries across Asia, Africa, Eastern Europe, Latin America, and Central America continue to face

unacceptably high rates of tuberculosis [3,4,12]. High-burden tuberculosis is prevalent among HIV-positive individuals, healthcare workers, and recent immigrants from tuberculosis-endemic regions in more developed countries. There is also evidence indicating an increased risk when using immunosuppressive medications, such as long-term corticosteroid therapy, which includes golimumab, etanercept, infliximab, and adalimumab. Patients taking any of these medications should have their tuberculosis status checked before and during treatment [15].Preventing and treating tuberculosis, an infectious disease, is possible. Nevertheless, in underdeveloped nations where providing adequate access to treatment remains a challenge, tuberculosis continues to be one of the leading causes of illness and mortality. Additional obstacles include lack of awareness, delays in diagnosis, difficulties in obtaining vaccinations, and non-adherence to prescribed medications. The WHO's recommended DOTS (Directly Observed Therapy) has shown significant efficacy in enhancing medication adherence among tuberculosis patients [12,19]. Additionally, a substantial decline in the prevalence of this infection has been attributed to immunization campaigns in developing nations [5,6,7].
Although the effectiveness of BCG vaccination in preventing tuberculosis is debated, numerous studies affirm that immunization plays a crucial role in combating the disease. Therefore, the emphasis on childhood immunization, particularly in underdeveloped nations, must be maintained [14]. Until we eradicate this disease globally, the WHO and other health organizations must continue funding the development of new strategies and research initiatives. It is imperative to create new anti-tuberculosis medications to effectively manage drug-resistant tuberculosis, treat latent tuberculosis infections, and simplify the treatment of tuberculosis caused by drug-susceptible organisms. Ashrafu and Zou [8] developed a vaccine distribution model with vaccination priority. However, the model has limitations as it overlooks the possibility that vaccinated individuals can still contract the disease if not entirely immune. In reality, vaccinations may significantly lower the risk of infection rather than completely eliminating it [5]. Therefore, the condition of incomplete immunity should be considered in infectious disease models. Reference [10] examined a basic SVIR epidemic model incorporating vaccination age, suggesting that the model allows vaccinated individuals to return to susceptibility as the vaccine's protective effects diminish. Building upon this, the SVIR infectious disease model and its integration techniques are further examined.

2. Mathematical Model

The collection of equations provided encompasses the four differential equations that illustrate the dynamics of a population concerning SVEI tuberculosis. These equations correspond to four distinct categories within the population: True Positive Susceptible (TPS), True Positive Vaccinated (TPV), True Positive Exposed (TPE), and True Positive Infected (TPI) [7].
TPS: Individuals in this category are susceptible to the illness but have not received the vaccination.
TPV: This group comprises individuals who have undergone the disease-prevention vaccination.
TPE: Individuals in this category have been exposed to the illness but have not yet contracted it.
TPI: Members of this group are individuals who have contracted the disease.
Equation (1) represents the rate at which TPS changes over time. It takes into account the total count of TPS, TPI, and TPV individuals. The constants b, φ, γ, and ν denote the rates of new individuals entering the TPS category, TPS individuals getting infected, TPS individuals receiving vaccinations, and TPV individuals returning to a susceptible status, respectively.
Equation (2) represents the rate of change of TPV over time, considering both the count of TPS and TPV individuals. The constants υ and γ symbolize the rates at which TPV individuals return to a susceptible state and new individuals enter the TPV category, respectively.

Equation (3) depicts the rate of change of TPE over time, taking into account the total count of TPS, TPI and TPE individuals. The constants φ, ε, and ν signify the rates at which new TPE individuals are added to the population, TPE individuals become infected, and TPV individuals get exposed, respectively.
Equation (4) represents the rate of change of TPI over time, factoring in the total count of TPI, TPE and TPV individuals. The parameter ζ signifies the rate of new infections, while the parameter ε symbolizes the rate of recovery from the disease for infected individuals.

$$\boldsymbol{TPS(t) = b - \varphi TPS(t)TPI(t) - (TPV + \gamma)TPS(t) + \nu TPV(t)} \quad \text{... (1)}$$

$$\boldsymbol{TPV(t) = \gamma TPS(t) - (TPV + \upsilon)TPV(t)} \quad \text{... (2)}$$

$$\boldsymbol{TPE(t) = \varphi TPS(t)TPI(t) - (TPV + \varepsilon)TPE(t)} \quad \text{... (3)}$$

$$\boldsymbol{TPI(t) = \zeta TPE(t) - (TPV + \varepsilon)TPI(t)} \quad \text{... (4)}$$

2.1 Equilibrium point of the model

The basic reproduction number represents the estimated number of newly infected individuals resulting from a single infected person within a fully susceptible population. Moreover, ψ0 is not a rate; rather it is a dimensionless quantity with units of time. It is widely understood that the disease-free equilibrium of an epidemic system is locally asymptotically stable when $R_0 < 1$, indicating the potential eradication of the disease from the population if the initial conditions of the variables TPS (t), TPV (t), TPE (t), and TPI (t) fall within the boundaries of the disease-free equilibrium. Additionally, when the equilibrium free from sickness is globally asymptotically stable, the disease can be eradicated from the population, regardless of the initial conditions and variables of the epidemic model. In the case of a population-wide pandemic, identifying scenarios where $R_0 < 1$ holds significant public health implications. System (9) allows a unique disease-free equilibrium point T0 = (TPS0, TPV0, 0, 0), where there is no disease present (TPE ≥ 0 and TPI ≥ 0).

Lemma:

If x_o is a DFE and fi(x) satisfies. Then the derivatives $D\mathcal{F}(x_o) and\ Dv(x_o)$ are partitioned as

$DF(x_o) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix} DV(x_o) = \begin{pmatrix} V & 0 \\ J_3 & J_4 \end{pmatrix}$ Where F and V are the m× m matrices defined by

$F = \left[\frac{\partial F_i}{\partial x_j}(x_o)\right]$ and $V = \left[\frac{\partial V_i}{\partial x_j}(x_o)\right]$ with 1≤i, j≤m

Further, F is non-negative, V is a non-singular M-matrix and all Eigen values of J_4 have positive real part.

$$\boldsymbol{F} = \begin{bmatrix} 0 & \varphi TPS \\ 0 & 0 \end{bmatrix}; \boldsymbol{V} = \begin{bmatrix} TPV + \zeta & 0 \\ -\zeta & TPV + \varepsilon \end{bmatrix} \quad \textbf{(5)}$$

Therefore, the reproduction number R_0 is given below

$$R_0 = \frac{b\varphi\zeta(TPV + \nu)}{TPV(TPV + \varepsilon)(TPV + \zeta)(TPV + \upsilon + \gamma)}$$

The substitution is mentioned in the above paragraph. Therefore, the endemic equilibrium points are given below

$$TPS^0 = \frac{(TPV + \varepsilon)(TPV + \zeta)}{\varphi\zeta}$$

$$TPV^0 = \frac{\gamma(TPV + \varepsilon)(TPV + \zeta)}{\varphi\zeta(TPV + \vartheta)}$$

$$TPI^0 = \frac{TPV(TPV + \vartheta + \gamma)}{\varphi(TPV + \vartheta)}(R_0 - 1)$$

$$TPE^0 = \frac{b}{TPV + \zeta}(1 - \frac{1}{R_0})$$

Theorem: T^0 was locally asymptotically stable on M if and only if $R_0 < 1$

Where M =(S,V,E,I) ∈ R^4:S(t)≥ 0, V(t)≥ 0, E(t) ≥ 0, I(t) ≥ 0 [6,15]

$$J = \begin{bmatrix} -\varphi(TPI) - TPV - \gamma & 0 & 0 & -\varphi(TPS) \\ \gamma & -TPV - \upsilon & 0 & 0 \\ \varphi TPI & 0 & -TPV - \varsigma & \varphi(TPS) \\ 0 & 0 & \varsigma & -TPV - \varepsilon \end{bmatrix}$$

Therefore, the disease free equilibrium is

$$J() = \begin{bmatrix} -TPV - \gamma & \nu & 0 & -\varphi\dfrac{b(TPV + \nu)}{TPV(TPV + \nu + \gamma)} \\ \gamma & -TPV - \nu & 0 & 0 \\ \varphi(TPI) & 0 & -TPV - \varsigma & \varphi\dfrac{b(TPV + \nu)}{TPV(TPV + \nu + \gamma)} \\ 0 & 0 & \varsigma & -TPV - \varepsilon \end{bmatrix}$$

$$DET(J(T^0) - \delta TPI) = 0$$

$$(\delta + TPV + \nu + \gamma)(\delta + TPV)(\delta^2 + (2TPV + \zeta + \varepsilon)\delta + (TPV + \zeta)(TPV + \varepsilon)(1 - R_0) = 0$$

$$\delta_1 = -(TPV + \nu + \gamma),\ \delta_2 = -(2TPV + \nu + \varepsilon)$$

$$\delta_{3,4} = \frac{-(2TPV + \zeta + \varepsilon) \pm \sqrt{(\zeta + \varepsilon)^2 + 4(TPV + \zeta)(TPV + \varepsilon)R_0 - \zeta_\varepsilon}}{2}$$

If $R_0 < 1$, it becomes evident that the eigen values of system (1-4) at the disease-free equilibrium are all real and negative. As a consequence, T_0, the equilibrium point devoid of sickness, achieves local asymptotic stability. This conclusion concludes the proof.

3. Numerical analysis of tuberculosis with high risk population

The initial conditions for the high-risk population are as follows:
S_0 = 9483 (initial susceptible population)
V_0 = 106 (initial vaccinated population)
E_0 = 807 (initial exposed population)
I_0 = 729 (initial infected population).
The parameters are as follows: b = 0.02 (birth rate), φ = 0.0001 (infection rate), δ = 0.01 (natural death rate), γ = 0.53 (vaccination rate), ν = 0.05 (loss of immunity rate), ζ = 0.005 (rate at which exposed individuals become infectious), and ε = 0.1 (recovery rate). Here, t_end represents duration from 1 to 7 days per week, and dt_end represents a duration of 24 hours per day. [1,10]

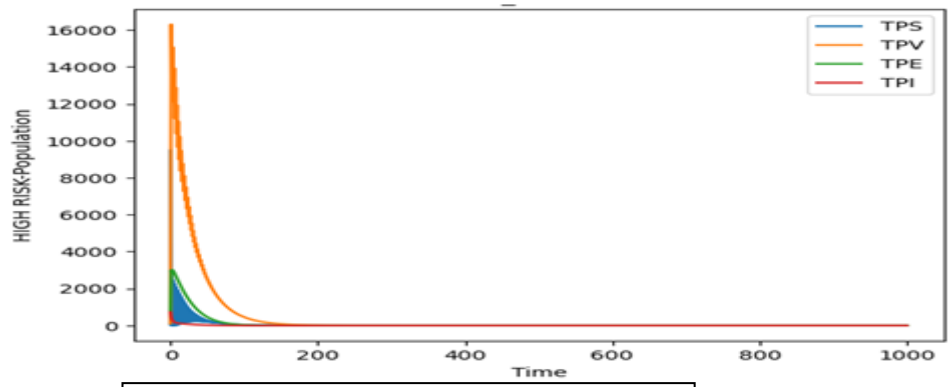

Figure 1 : Solution for t_end = 100 , dt = 0.1

True Positive Susceptible (TPS): This line represents the population susceptible to the disease. It starts at a high level and decreases over time suggesting individuals either receiving vaccinations or getting exposed or infected as time progresses.
True Positive Vaccination (TPV): This line represents the vaccinated population. It begins at a low level and increases over time reflecting the success of a vaccination campaign.
True Positive Exposed (TPE): This line indicates the population exposed to the disease. It starts at a low level and rises over time signifying the spread of the disease within the population.
True Positive Infected (TPI): This line represents the population infected with the disease. It also starts at a low level and increases over time indicating a rise in infections.

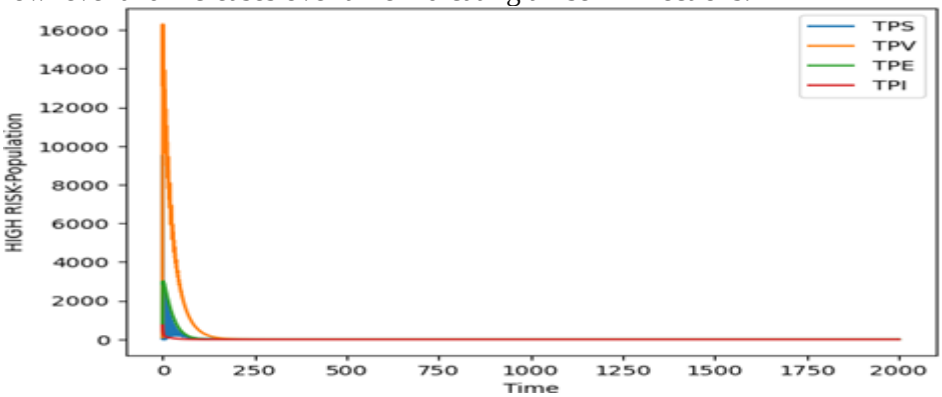

Figure 2: Solution for t_end = 200, dt = 0.1

True Positive Susceptible (TPS): The blue line representing the susceptible population exhibits a sharp peak at the beginning, followed by a rapid decrease. This pattern might indicate a swift spread of the disease initially, succeeded by a decline as individuals transition into other categories.

True Positive Exposed (TPE): The orange line representing the exposed population displays a more gradual decline. This trend suggests a slower reduction in exposure to the disease over time.
True Positive Infected (TPI): The green line representing the infected population shows a slight increase initially followed by a gradual decrease. This might imply an initial phase of infection spread succeeded by a gradual recovery or movement into other categories.

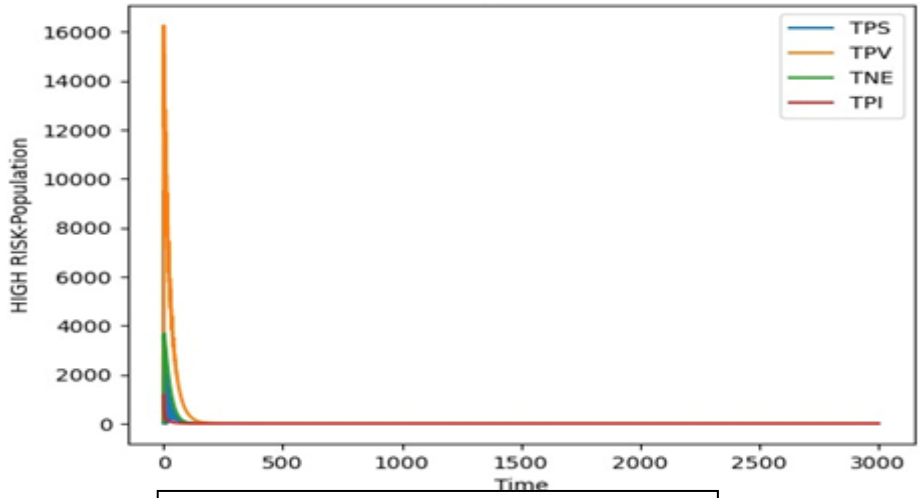

Figure 3: Solution for t_end = 300, dt = 0.1

True Positive Susceptible (TPS): The blue line depicting the susceptible population initially shows a steep drop followed by stabilization. This suggests that after a rapid decrease the number of susceptible individuals remains steady.
True Positive Exposed (TPE): The orange line representing the exposed population maintains a relatively consistent level throughout the graph. This may indicate that the number of exposed individuals remains constant over this time period.
True Positive Infected (TPI): The green line illustrating the infected population also remains relatively constant throughout the graph. This could indicate that the number of infections stabilizes over this time period.

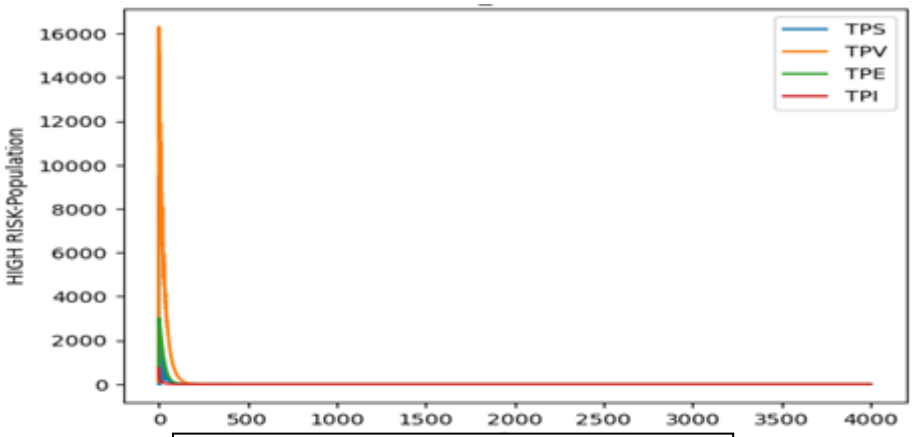

Figure 4: Solution for t_end = 400, dt = 0.1

True Positive Susceptible (TPS): The orange line illustrating the susceptible population initially displays a steep drop, followed by stabilization. This may indicate that after a rapid decrease the number of susceptible individuals reaches a stable level.
True Positive Exposed (TPE): The blue line representing the exposed population begins at the bottom and steadily rises until it reaches the top. This suggests that the number of exposed individuals increases throughout this time period.

True Positive Infected (TPI): The green line depicting the infected population also starts at the bottom and gradually increases until it reaches the top. This indicates an increase in the number of infections over this time period.

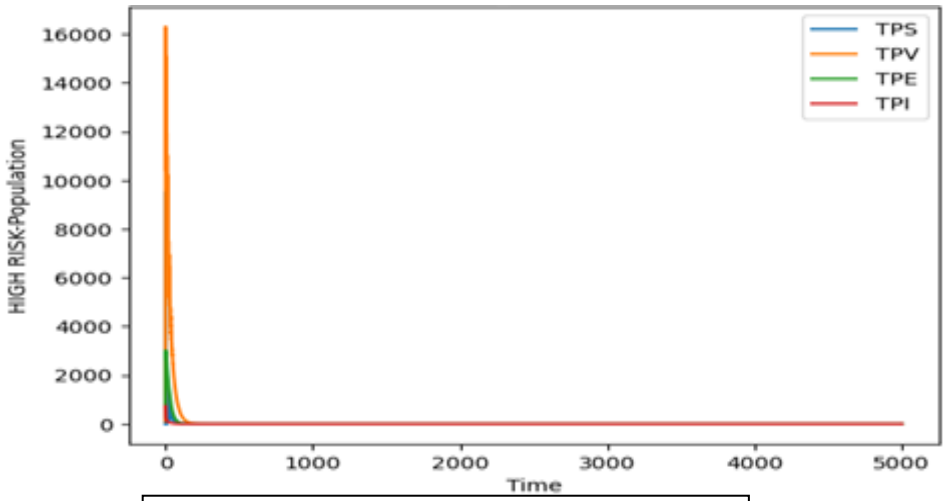

Figure 5 : Solution for t_end = 500 , dt = 0.1

True Positive Susceptible (TPS): The blue line representing the susceptible population displays a sharp increase initially followed by stabilization. This pattern suggests a rapid rise in susceptibility at the outset succeeded by stabilization.
True Positive Exposed (TPE): The orange line illustrating the exposed population shows a gradual increase and then levels off. This pattern implies a slow increase in exposure to the disease over time followed by stabilization.
True Positive Infected (TPI): The green line depicting the infected population exhibits a sharp decrease initially and then levels off. This indicates an initial decline in infections followed by stabilization.

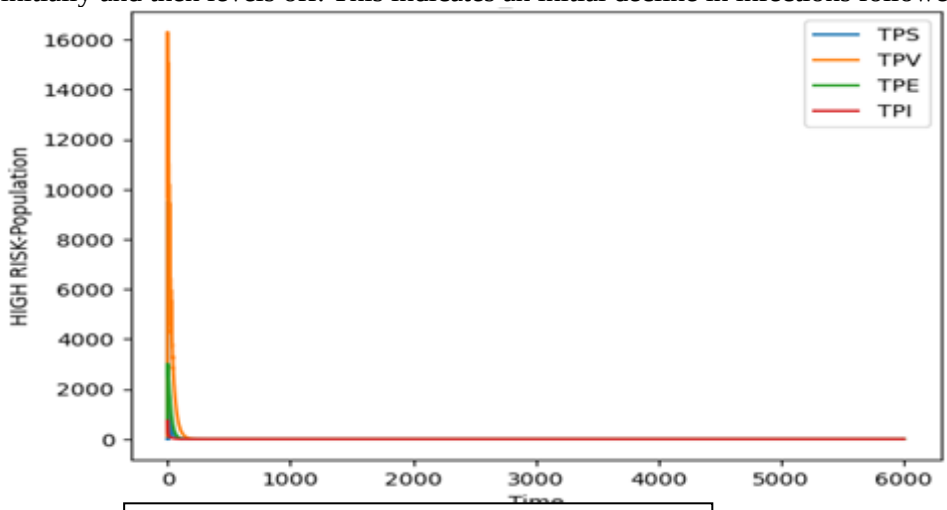

Figure 6 : Solution fort_end = 600 , dt = 0.1

True Positive Susceptible (TPS): The blue line representing the susceptible population exhibits a particular trend over time. Without additional context or data points providing a detailed interpretation of this trend are challenging.
True Positive Exposed (TPE): The orange line representing the exposed population also shows a specific trend over time. Similarly, without additional context or data points it's challenging to provide a detailed interpretation of this trend.
True Positive Infected (TPI): The green line depicting the infected population displays a distinct trend over time. Again, without additional context or data points it's challenging to provide a detailed interpretation of this trend.

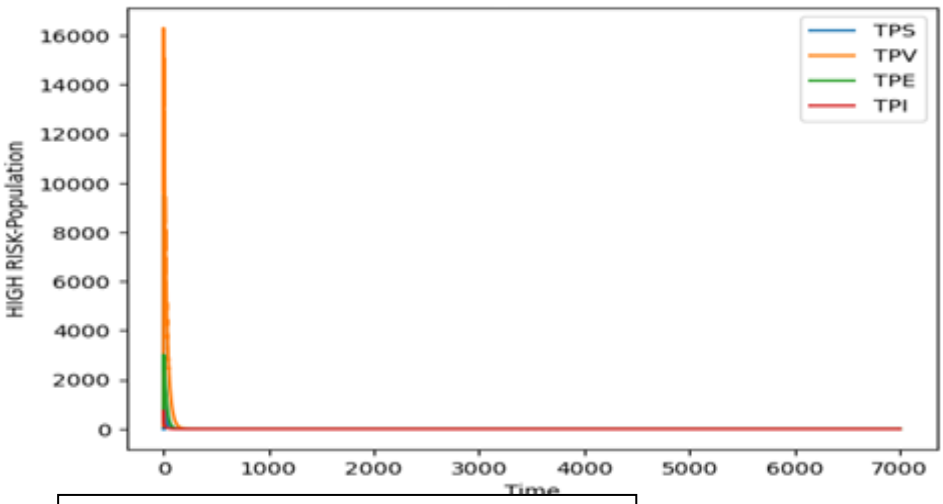

Figure 7 : Solution fort_end = 700, dt = 0.1

True Positive Susceptible (TPS): The blue line representing the susceptible population appears as a straight line starting from the top left corner and ending at the bottom right corner of the graph. This suggests a consistent decrease in susceptibility over time.
True Positive Exposed (TPE): The orange line representing the exposed population displays a curved line starting from the top left corner and ending at the bottom right corner of the graph. This pattern indicates a non-linear decrease in exposure over time.
True Positive Infected (TPI): The green line representing the infected population is depicted as a straight line starting from the bottom left corner and ending at the top right corner of the graph. This could suggest a steady increase in infections over time.

4. Numerical Analysis of Tuberculosis with Low Risk Population

The initial conditions for the Low-risk population are as follows:
S_0 = 9483 (initial susceptible population)
V_0 = 106 (initial vaccinated population)
E_0 = 807 (initial exposed population)
I_0 = 64 (initial infected population)

The parameters are as follows: b = 0.02 (birth rate), φ = 0.0001 (infection rate), δ = 0.01 (natural death rate), γ = 0.53 (vaccination rate), ν = 0.05 (loss of immunity rate), ζ = 0.005 (rate at which exposed individuals become infectious), and ε = 0.1 (recovery rate).Here t_end represents a duration from 1 to 7 days per week and dt_end represents a duration of 24 hours per day. [13]

The solution for t_end =100, dt = 0.1 illustrates the relationship between the low risk population and time. The graph displays three lines of distinct population such as TPS, TPE and TPI. The TPS line depicted in blue exhibits a sharp peak around t=100 signifying a significant change in the TPS population at that specific point in time. The TPE line marked in orange and the TPI line shown in green both remain relatively flat suggesting that the TPE and TPI populations are less affected by changes over time. This implies that the TPS population is more time-sensitive compared to the TPE and TPI populations. It's important to note that without additional context or data this interpretation remains at a high level.

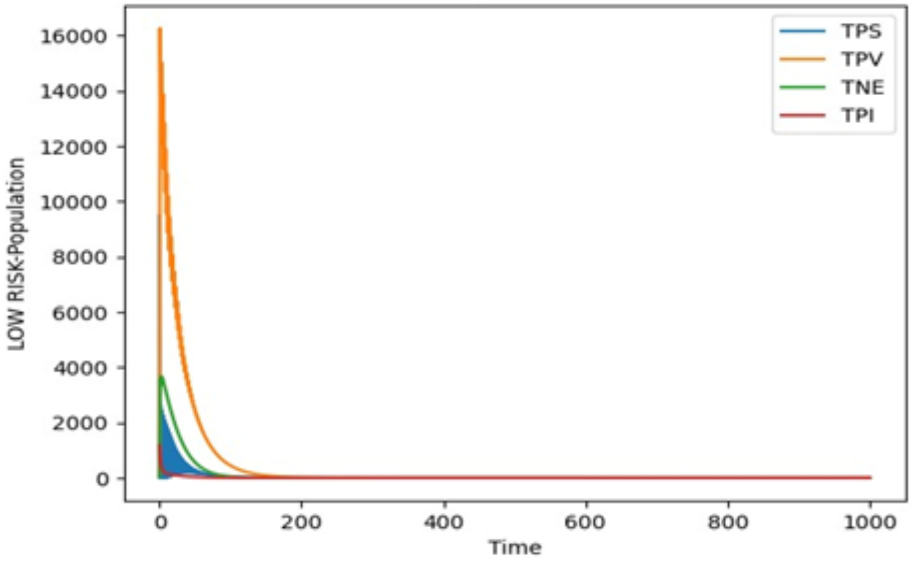

Figure 8 : Solution for t_end = 100 and dt = 0.1

The graph depicts the relationship between the low risk population and time. The graph features three lines: TPS, TPE and TPI with t_end = 200 and dt = 0.1. All three lines (TPS, TPE, and TPI) demonstrate a decreasing trend as time progresses, signifying a reduction in the low risk population over time. The TPS line is represented in blue, the TPE line in orange, and the TPI line in green. It's important to note that this interpretation is at a high level and might require additional context or data for a more accurate understanding. Further information regarding what each population (TPS, TPE and TPI) represents would be beneficial for a more precise analysis. Please consider that this interpretation assumes the y-axis represents the low risk population, while the x-axis represents time.

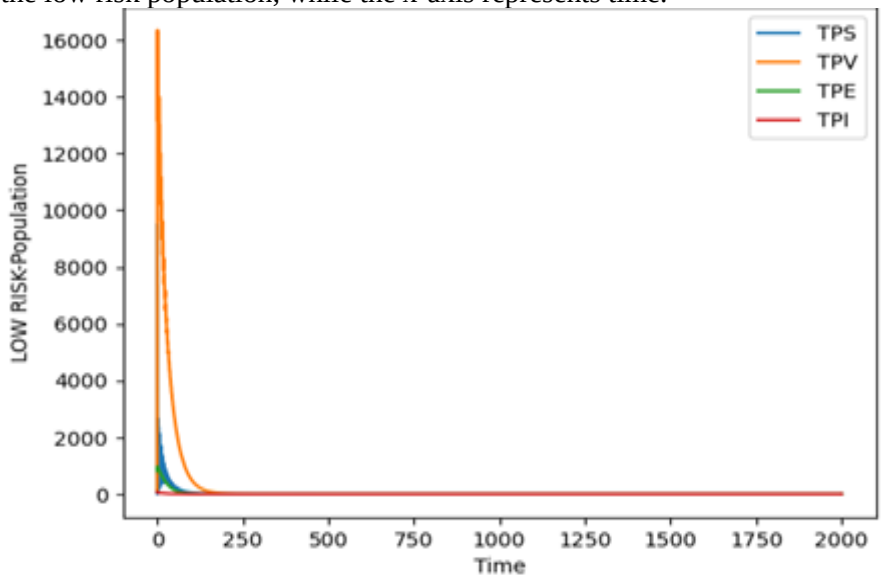

Figure 9 : Solution for t_end = 200, dt = 0.1

The graph illustrates the relationship between the low-risk population and time. It consists of three lines: TPS, TPE and TPI with parameters set as t_end = 300 and dt = 0.1. The TPS line depicted in blue registers the highest value initially but subsequently it decreases rapidly and stabilizes. In contrast the TPE line marked in orange and the TPI line shown in green both start at lower levels. However, they steadily increase over time and eventually plateau. This trend suggests that the TPS population experiences a rapid decline over time while the TPE and TPI populations exhibit a gradual increase.

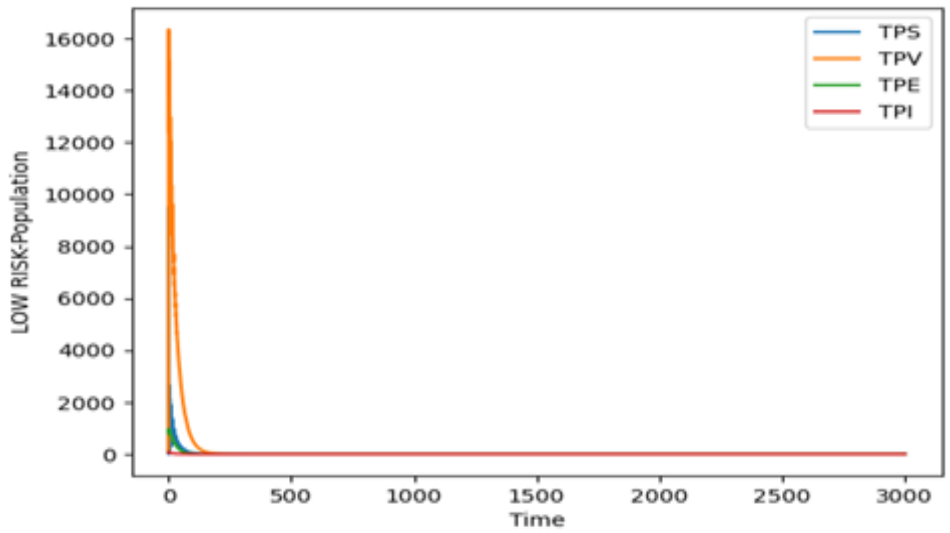

Figure 10 : Solution for t_end = 300, dt = 0.1

This graph illustrates the relationship between the low-risk population and time. It features three lines: TPS, TPE and TPI with parameters set at t_end = 400 and dt = 0.1. The TPS line represented in blue begins at a high value but then experiences a rapid decrease. In contrast the TPE line is depicted in orange and the TPI line in green. Both lines start at a low value but demonstrate a rapid increase. Towards the end of the graph the TPS line concludes at a low value while the TPE and TPI lines conclude at a high value. This pattern indicates that the TPS population decreases rapidly over time while the TPE and TPI populations increase rapidly.

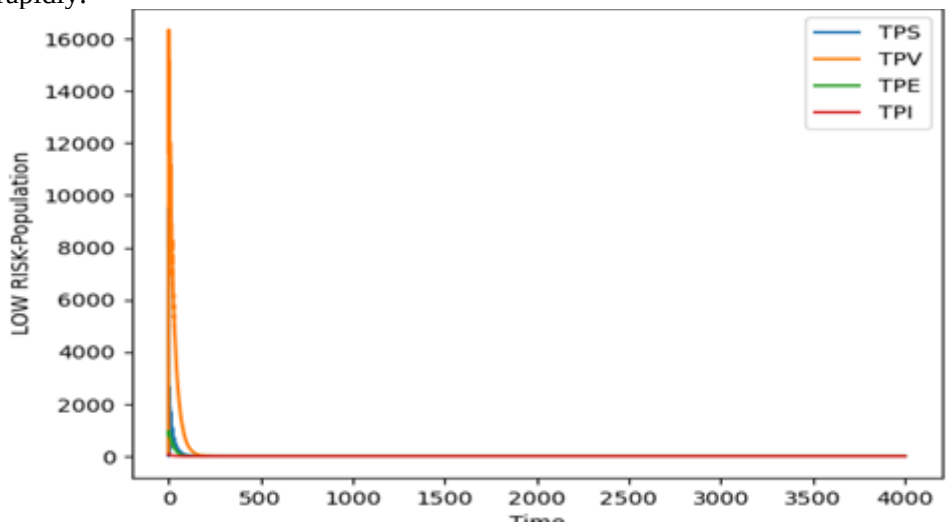

Figure 11 : Solution for t_end = 400, dt = 0.1

This graph illustrates the relationship between the low-risk population and time. It consists of three lines: TPS, TPE and TPI with the parameters set at t_end = 500 and dt = 0.1. The TPS line is blue while the TPE line is orange. Both these lines depict a decrease as time progresses suggesting a reduction in these populations over time. Conversely the TPI line shown in green indicates an increase over time signifying a growth in this population. This observation implies that the TPS and TPE populations decrease over time while the TPI population increases.

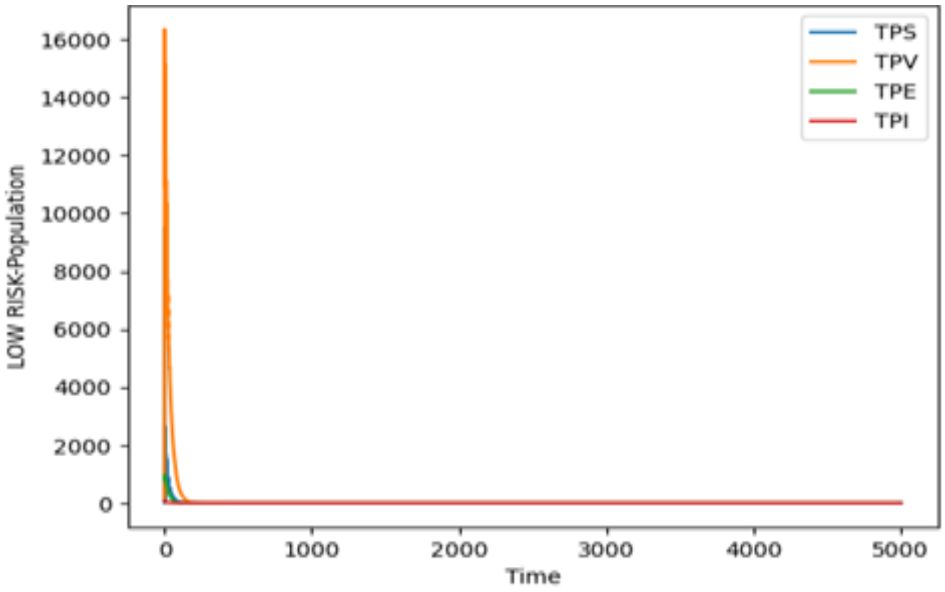

Figure 12 : Solution for t_end = 500, dt = 0.1

The solution is presented for t_end = 600, dt = 0.1 showcasing the relationship between time and the low-risk population across three different variables: TPS, TPE and TPI. The TPS line marked in blue and the TPE line in orange both exhibit a linear relationship with time. Meanwhile the TPI line depicted in green demonstrates an exponential relationship with time.

This observation suggests that the TPS and TPE populations change linearly over time, while the TPI population changes exponentially. It's important to note that without additional context or data, this interpretation remains high-level. To gain a more accurate understanding, additional information about what each population TPS, TPE, and TPI represents would be beneficial.

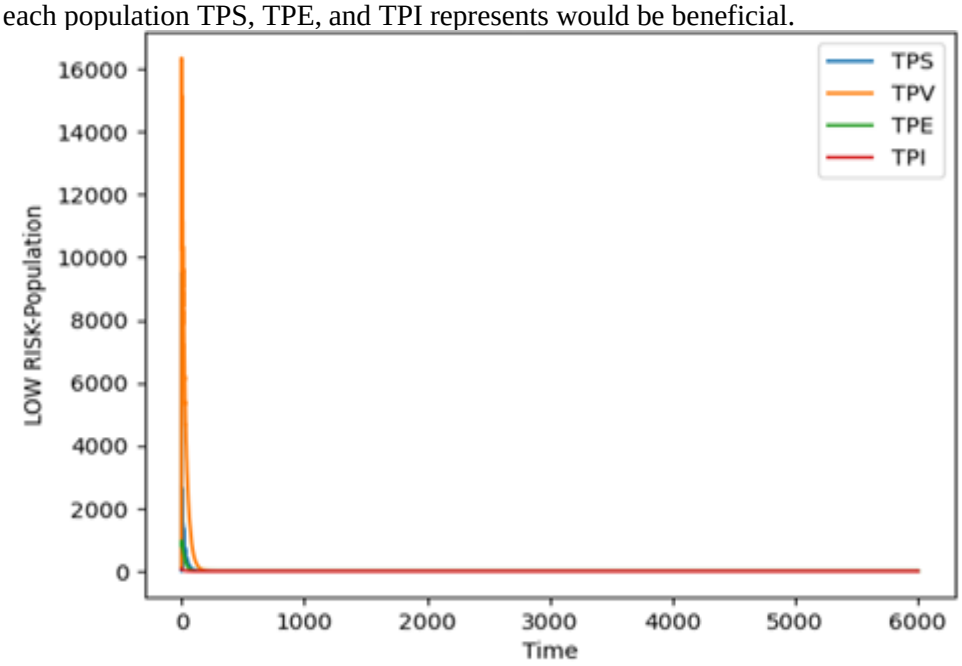

Figure 13: Solution for t_end = 600,dt = 0.1

The graph illustrates the relationship between the low risk population and time with three labeled lines namely TPS, TPE and TPI where the x-axis represents time and the y-axis represent the low risk population. Both TPS and TPE lines appear constant suggesting no change in these populations over time. In contrast the TPI line shows a decrease as time progresses indicating a reduction in the low risk population within this category over time. Additionally, a red vertical line is visible at time=0 potentially marking the beginning of observation.

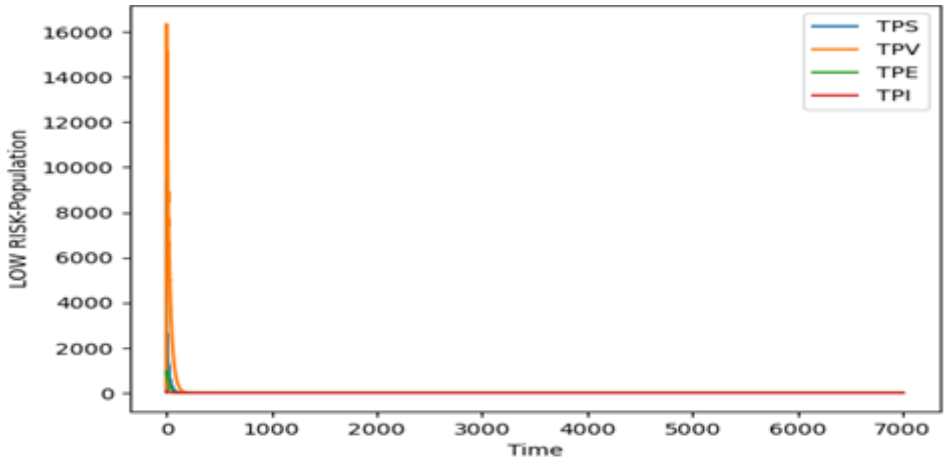

Figure 14 : Solution for t_end = 700, dt = 0.1

5. Conclusion:

High Risk Population graph illustrates changes in the population of a specific disease over time. The blue line represents the susceptible population, the orange line represents the exposed population, the green line represents the infected population and the yellow line represents the vaccinated population. The graph suggests that the disease initially spread rapidly but the number of susceptible individuals decreased over time. The number of exposed individuals remained relatively consistent throughout the graph, while the number of infected individuals showed a slight increase initially followed by a gradual decrease. The success of the vaccination campaign is reflected in the increasing trend of the vaccinated population over time.
Furthermore, we observe that susceptibility and exposure steadily decline over time, while infections steadily rise. The graph is composed of three lines labeled TPS, TPE and TPI. Time is represented on the x-axis, while the low risk population is represented on the y-axis. The TPS and TPE lines seem to exhibit continuous variations suggesting that these populations do not fluctuate significantly over time. In contrast, the TPI line declines with increasing time, indicating a gradual decrease in the population classified as low risk within this category.

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