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Crossing Numbers of the Cartesian Product of the Double Triangular Snake Graphs With Path P_m.

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Abstract

The crossing number Cr(G) of a graph G is the least number of edge crossings in all possible good drawings of G in the plane. Join and Cartesian products of graphs have many interesting graph-theoretical properties. In this paper, we evaluate the crossing number of the Cartesian product of double triangular snake graph DT_2 with the path P_m. In this paper, we proved $Cr(DT_2 \times P_m) = 6(m - 2)$, form ≥ 2 where Cr denotes the crossing number.

Keywords: Crossing number, Cartesian product, Join product, Double triangular snake graph, Path, cycle.

Mathematics Subject Classification (2010): 68R10; 05C10; 05C62

Introduction

The crossing number of a graph G is the least possible number of edge crossings in any drawing of G in the plane. It is denoted by $Cr(G)$. A drawing of a graph is good if every two edges have at most one point in common, which is either a common end vertex or a crossing. We denote the number of crossings of a graph G in a drawing τ in the plane by $Cr(G)$. For two subgraphs H and H' of G such that $E(H) \cap E(H') \neq \phi$, $Cr_\tau(H, H')$ denotes the edge crossings between E(H) and E(H') in the drawing τ. The union of the two graphs F and K, denoted by $F \cup K$ and it is the graph with vertex set $V(F) \cup (K)$ and edge set $E(F) \cup E(K)$. The join of the two graphs F and K, denoted by $F + K$, is the graph obtained from $F \cup K$ by joining every vertex of F to every vertex of K.
The Cartesian product of the two graphs F and K, denoted by $F \times K$, is the graph whose vertex set is $V(F) \times V(K)$ and the edges are of the form $(a, b)(m, n)$ where either $a = m$ and $bn \in E(K)$ or $b = n$ and $am \in E(F)$.
Let P_m denote the path on m vertices of length (m-1). There are few results with $Cr(G \times P_m)$, where G is a graph on 7 vertices. Finding the value of $Cr(G \times P_m)$ has been investigated in [1], [2], [3], [5], [6], [7], [8], [9], [10]. In this paper, we evaluated the exact value of $Cr(DT_2 \times P_m)$ for $m \geq 2$.

1 Crossing numbers of the Cartesian prod- uct of Double Triangular snake graph DT_2 with P_m.

Definition 2.1. (Double Triangular Snake graph DT_p**)** A graph DT_p with p blocks is obtained from a path on vertices α_0, α_1, α_2,, α_p and by joining α_i and α_{i+1} to two new vertices α_{p+1+i} and α_{2p+1+i} for i= 0,1, 2, ..., p-1

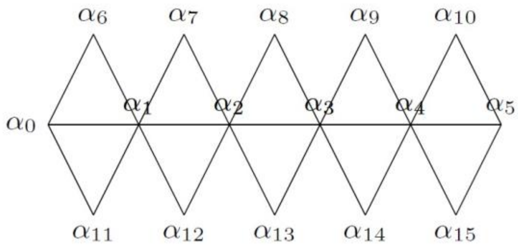

Figure 1: (Double Triangular snake with 5 blocks)

Drawing of $DT_2 \times P_m$**:**

For $m \geq 2$, Graph $DT_2 \times P_m$ has 7m vertices and $(17m - 7)$ edges that are the edges in the m copies of graph DT_2 and in the 7 paths of m vertices.

Notation:

(1) $(DT_2)^i$: i^{th} copy of Double Triangular Snake graph DT_2.

(2) $(E_2)^i$: Edges which join vertices of $(i + 1)^{th}$ copy of Double Tri- angular Snake graph DT_2 with corresponding vertices of i^{th} copy of the graph DT_2.

(3)

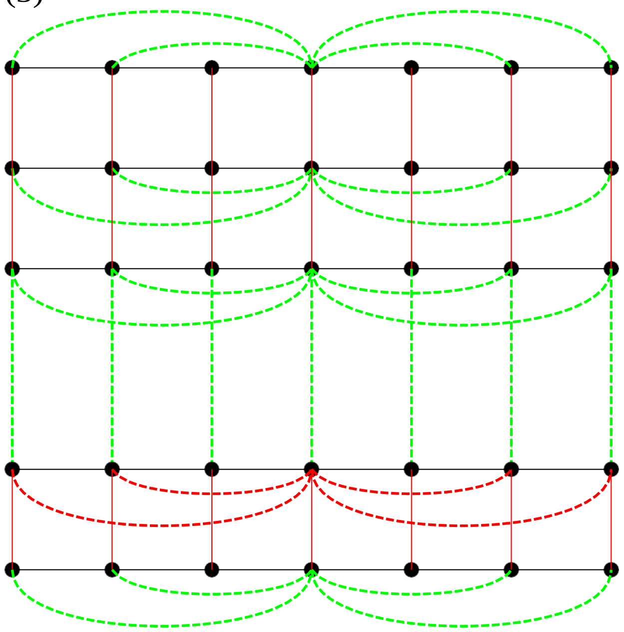

Figure 2: ($DT_2 \times P_m$.)

Remark 2.1. $Cr(DT_2 \times P_2) = 0$.

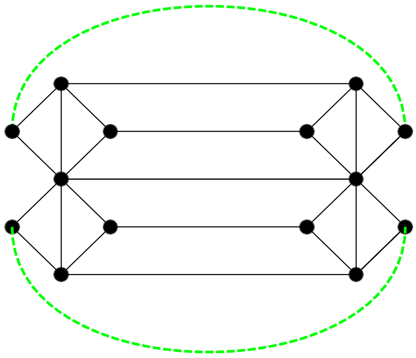

Figure 3: ($DT_2 \times P_2$)

Lemma 2.1. Let $Z(n,m)=\left\lfloor\frac{n}{2}\right\rfloor\cdot\left\lfloor\frac{n-1}{2}\right\rfloor\cdot\left\lfloor\frac{m}{2}\right\rfloor\cdot\left\lfloor\frac{m-1}{2}\right\rfloor$

(i) $Cr(K_{n,m})=Z(n,m)$ where $\min\{m, n\} \leq 6$,.

(ii) $Cr(K_{n,m}) \leq Z(n, m)$ for $m, n \in N$.

Theorem 2.1. $Cr_{\tau}(DT_2 + 2K_1) = 6$.

Proof. $DT_2 + K_1$ is planar. Consider a good drawing of the graph $DT_2 + 2K_1$ given in **fig. 4**.

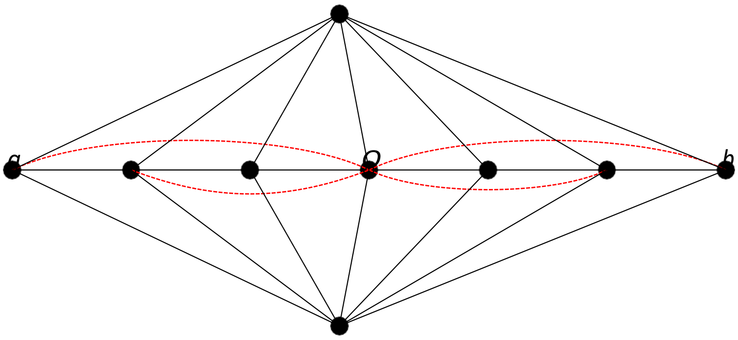

Figure 4: ($DT_2 + 2K_1$)

From fig. 4 we can conclude that,

$Cr_{\tau}(DT_2 + 2K_1) \leq 6$ (1)

Let us prove the otherway inequality.

Any drawing τ of the graph $DT_2 + 2K_1$ contains a sub drawing of $K_{3,6}$. Thus $Cr_{\tau}(DT_2 + 2K_1) \geq Cr_{\tau}(K_{6,3}) = 6$.

$Cr_{\tau}(DT_2 + 2K_1) \geq 6$ (2)

form **eq. (1)** and **eq. (2)**, we can conclude that,

$Cr_{\tau}(DT_2 + 2K_1) = 6$

Lemma 2.2. For any good drawing of the graph $DT_2 \times P_2$, let y be a vertex such that $y \notin V(DT_2)$ and graph S obtained by joining all vertices of one copy of the graph DT_2 with the vertex y then $Cr_{\tau}1$ (S) =6.

Proof. Consider a good drawing of the graph S, as shown in **fig. 5**.

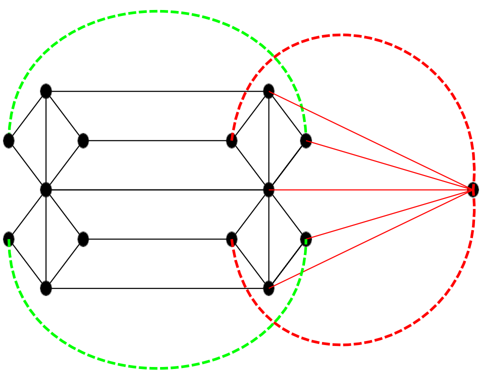

Figure 5: ((Drawing of the graph S)

Thus from **fig. 5** we can conclude that,

$Cr_{\tau_1}(S) \leq 6$.

Now we have to prove other way inequality. By contracting all vertices and edges of another copy of the graph DT_2 in any drawing τ_1 of graph S, we get a contraction of subdrawing of the graph $DT_2 + 2K_1$. Thus,

$Cr_{\tau_1}(S) \geq Cr_{\tau_1}(DT_2 + 2K_1) = 6$.

$Cr_{\tau_1}(S) \geq 6$. Hence proved.

Theorem 2.2. $Cr_{\tau}(DT_2 \times P_3) = 6$.
Proof. Consider a good drawing of the graph $DT_2 \times P_3$ as shown in **fig. 6.**

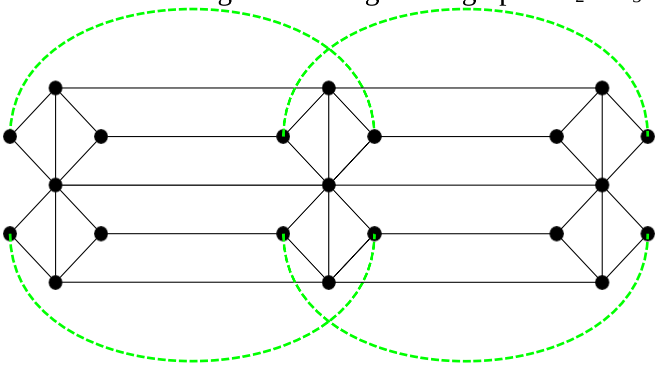

Figure 6: ($DT_2 \times P_3$**)**

Thus from **fig. 6** we can conclude that,
$Cr_{\tau}(DT_2 \times P_3) \leq 6$
Now we have to prove otherway inequality. By contracting all vertices and edges of 3^{rd} copy of the graph DT_2 in any drawing τ of the graph $DT_2 \times P_3$, we get a contraction of the graph S. Thus,
$Cr_{\tau}(DT_2 \times P_3) \geq Cr_{\tau}(S) = 6$
$Cr_{\tau}(DT_2 \times P_3) \geq 6$.
Hence proved.

Theorem 2.3. $Cr_{\tau}(DT_2 \times P_m) = 6(m - 2)$, for $m \geq 2$.
Proof. We prove this by the method of induction on m. By **theorem 2.2** result holds for m=3. Let us assume the result holds for less than m. Now we have to prove the result for m. In any good drawing τ of the graph $DT_2 \times P_m$, by Contracting all vertices and edges of n^{th} copy of the graph DT_2. We get a drawing of a disjoint union of subgraphs $DT_2 \times P_{m-1}$ and S. Thus
$Cr_{\tau}(DT_2 \times P_m) \geq Cr_{\tau}(DT_2 \times P_{m-1}) + Cr_{\tau}(S) = 6(m-3)+6 = 6(m-2)$
$Cr_{\tau}(DT_2 \times P_m) \geq 6(m - 2)$
and from the drawing of the graph $DT_2 \times P_m$ shown in **fig. 2**, we can conclude that,
$Cr_{\tau}(DT_2 \times P_m) \leq 6(m - 2)$
Hence proved.

2 Conclusion

In this paper, we obtained the exact crossing number of the Cartesian product of double triangular snake graph DT_2 with P_m.

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5 Data availability statement

Data sharing is not applicable to this article as no datasets were gen- erated or analyzed during the current study.

6 Declarations

This declaration is not applicable.

7 Conflict of interest

There is no any conflict of interest.

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