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Reduction of Boundary Value Problem using Shape Function

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***Abstract*:**

This research looks at the MHD flow of a power-law fluid on a stretched sheet with a uniform heat source. The boundary shape function technique translated the resulting Couple of Nonlinear Ordinary Differential equations (BVP) with boundary conditions into a related initial value problem (IVP). The BVP's solution is represented by the boundary shape function (BSF), and a further new variable is the free function. With the right method, the initial value of the problem may be numerically solved.'

Key Words: Power Law Surface Temperature, Power Law Heat Flux, Boundary Shape Function Method, Initial Value Problem

Introduction:

The study of the magnetic appears and dynamics of electrically conducting fluids is known as magnetohydrodynamics or MHD. Electrolytes, salt water, liquid metals, and plasmas are characteristics of these types of magnetofluids. Magnetically driven hydrodynamics (MHD) is based on the theory that magnetic fields may generate currents in a continuing conductive fluid, polarizing the stream and altering the magnetic field. Laminar boundary flow of an electrically conducting fluid over a moving continuous stretching surface is essential in many manufacturing processes, such as those that produce materials through polymer extrusion, continuous plastic film stretching, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion, and spinning, cooling of metallic sheets, or electronic chips..

Stretching a surface in a stationary cooling fluid is a crucial step in many production processes, including those that make plastic sheets and glass. Many researchers are motivated to investigate the flow and heat transfer of the boundary layer created above the stretched surface because of the problem's straightforward geometry. The following are the fundamental distinctions between heat/mass transfer and flow past a stretched and decreasing sheet: Various kinds of decreasing velocities are considered. The properties of heat or mass transfer for flow over several kinds of shrinking sheets are given.

The physical characteristics of the final product are highly dependent on the cooling velocity during present metallurgical and metal-working operations, such as drawing continuous filaments through quiescent fluids and annealing and tinning copper wires. Investigating the magnetohydrodynamic (MHD) movement of an electrically conducting fluid is hence extremely amazing.[1]

Boundary value problems (BVPs) are used in a number of fields, such as engineering, thin-film theory, aero-elasticity, theory of control, sandwich beam analysis, deflection of beam theory, electromagnetic waves, and incompressible flows. [2] They are also used in the boundary layer of fluid mechanics. The BVP with multipoint boundary conditions [3]and the singly perturbed BVP with Robin boundary conditions[4] were both solved using boundary shape function theory. The boundary conditions for the third-order three-point nonlinear BVP are challenging to successfully meet. There are so many methods to convert BVPs to IVPs like linear group transformation [5], differential transformation method[6], systematic deductive group method[7], shooting method[8], etc. Unless the algorithm is specifically created to satisfy every boundary condition. The research presents an algorithm that automatically satisfies the three-point boundary conditions to solve nonlinear third-order three-point boundary value problems (BVPs). In this paper, the approach is built on the innovative idea of a boundary shape function[9,10]. The function known as the boundary shape function (BSF) is designed to automatically satisfy the specified boundary criteria. Based on the boundary shape functions, an effective method is developed for solving nonlinear third-order ordinary differential equations.

Methodology:

We will examine an ordinary differential equation (ODE) with third-order nonlinearity.

$$y''' = F(z, y(z), y'(z), y''(z)), \quad 0 < z < l \tag{1}$$

With independent three-point boundary conditions

$$c_{11}y(0) + c_{12}y'(0) + c_{13}y''(0) = b_1 \tag{2}$$

$$c_{21}y(a) + c_{22}y'(a) + c_{23}y''(a) = b_2 \tag{3}$$

$$c_{31}y(l) + c_{32}y'(l) + c_{33}y''(l) = b_3 \tag{4}$$

Theorem : For any free function $f(z) \in C^2[0, l]$, if $s_k(z), k = 1, 2, 3,$ satisfy then the boundary shape function B(z), given by

$$\begin{aligned} B(z) &= f(z) + s_1(z)[b1 - c_{11}f(0) - c_{12}f'(0) - c_{13}f''(0)] \\ &+ s_2(z)[b_2 - c_{21}f(a) - c_{22}f'(a) - c_{23}f''(a)] \\ &+ s_3(z)[b_3 - c_{31}f(l) - c_{32}f'(l) - c_{33}f''(l)] \end{aligned} \tag{5}$$

Is existent and satisfies the separated boundary condition

$$c_{11}B(0) + c_{12}B'(0) + c_{13}B''(0) = b_1 \tag{6}$$

$$c_{21}B(a) + c_{22}B'(a) + c_{23}B''(a) = b_2 \tag{7}$$

$$c_{31}B(l) + c_{32}B'(l) + c_{33}B''(l) = b_3 \tag{8}$$

Mathematical Analysis:

To obtained couple of nonlinear ordinary differential equations are given by

Power law surface temperature

$$n(-f'')^{n-1}f''' - f'^2 + \frac{2n}{n+1}ff' - Mf' - k_pf' - Ff'^2 = 0 \tag{1}$$

$$(1 + \varepsilon\theta)\theta'' + P_r\left\{\left(\frac{2n}{n+1}\right)f\theta' - \lambda f'\theta\right\} + E_cP_rMf'^2 + P_r(1 + \varepsilon\theta)(\alpha f' + \beta\theta) + \varepsilon\theta'^2 = 0 \tag{2}$$

$$f = 0,\ f' = 1,\ \theta = 1\ at\ \eta = 0\ \ \&\ \ f' \to 0,\ \theta \to 0\ as\ \eta \to \infty \tag{3}$$

Power -law heat flux

$$n(-f'')^{n-1}f''' - f'^2 + \frac{2n}{n+1}ff' - Mf' - k_pf' - Ff'^2 = 0 \tag{4}$$

$$(1 + \varepsilon\phi)\phi'' + P_r\left\{\left(\frac{2n}{n+1}\right)f\phi' - \lambda f'\phi\right\} + E_cP_rMf'^2 + P_r(1 + \varepsilon\phi)(\alpha^* f' + \beta^*\phi) + \varepsilon\phi'^2 = 0 \tag{5}$$

$$f = 0,\ f' = 1,\ \phi' = -1/(1 + \varepsilon\phi)\ \ at\ \eta = 0\ \ \&\ f' \to 0,\ \phi \to 0\ as\ \eta \to \infty \tag{6}$$

Where the primes denote the differentiation with respect to η

$$P_r = \frac{\rho C_p u_w x}{k_\infty (Re_x)^2/(n+1)},\ E_c = \frac{c^2 L^2}{C_p(T_w - T_\infty)}\ and\ F = \frac{c_b x}{\sqrt{k_p^*}}$$

Solution:

By using boundary condition for equation (1) to (3), take three shape function for $f(\eta)$ satisfying

$$s_1(0) = 1,\ \ s_1'(0) = 0,\ \ s_1'(\infty) = 0, \tag{7}$$

$$s_2(0) = 0,\ \ s_2'(0) = 1,\ \ s_2'(\infty) = 0, \tag{8}$$

$$s_3(0) = 0,\ \ s_3'(0) = 0,\ \ s_3'(\infty) = 1, \tag{9}$$

Hence, we have

$$s_1(\eta) = e^{-\eta^2},\qquad s_2(\eta) = -\eta e^{-\eta},\qquad s_3(\eta) = \eta - 1 + e^{-\eta}$$

Then we consider the variable transformation from $f(\eta)$ to $u(\eta)$ by

$$f(\eta) = u(\eta) - s_1(\eta)u(0) - s_2(\eta)[u'(0) - 1] - s_3(\eta)u'(\infty) \tag{10}$$

Hence, f(η) in Eq. (5) is a boundary shape function, which for any function F(η) automatically satisfies the boundary conditions:

$$f(0) = 0,\ \ f'(0) = 1,\ \ f'(\infty) = 0 \tag{11}$$

Now by substituting the $u(\eta)$ by $f(\eta)$, we get

$$\left(F\left(u' + 2\eta e^{-\eta^2}u(0) - (1 - e^{-\eta})u'(\infty) - (e^{-\eta} - \eta e^{-\eta})(u'(0) - 1)\right) + K + M - 1\right)\left(u' + 2\eta e^{-\eta^2}u(0) - (1 - e^{-\eta})u'(\infty) - (e^{-\eta} - \eta e^{-\eta})(u'(0) - 1)\right) + n\left((u'' + u(0)(2\eta e^{-\eta^2} - 4\eta^2 e^{-\eta^2}) - e^{-\eta}u'(\infty) - (\eta e^{-\eta} - 2e^{-\eta})(u'(0) - 1)\right)^{n-1}\left(u''' - u(0)e^{-\eta^2}(12\eta - 8\eta^3) + e^{-\eta}u'(\infty) - (3e^{-\eta} - \eta e^{-\eta})(u'(0) - 1)\right) + \frac{2n}{n+1}\left(u - u(0)e^{-\eta^2} - (\eta - 1 + e^{-\eta})u'(\infty) - \eta e^{-\eta}(u'(0) - 1)\right)\left((u'' + u(0)(2\eta e^{-\eta^2} - 4\eta^2 e^{-\eta^2}) - e^{-\eta}u'(\infty) - (\eta e^{-\eta} - 2e^{-\eta})(u'(0) - 1)\right) = 0 \tag{12}$$

Now take $u(0) = 0$ & $u'(0) = 1$ in equation (12)

$$\left(F\left(u' - (1 - e^{-\eta})u'(\infty)\right) + K + M - 1\right)\left(u' - (1 - e^{-\eta})u'(\infty)\right) + ((u'' - e^{-\eta}u'(\infty))^{n-1}\left(u''' + e^{-\eta}u'(\infty)\right) + \frac{2n}{n+1}\left(u - (\eta - 1 + e^{-\eta})u'(\infty)\right)((u'' - e^{-\eta}u'(\infty)) = 0 \tag{13}$$

Now we reduce the given equ.(13) in system of first order ODEs take $u = u_1,\ u' = u_2,\ u'' = u_3$ and put in equation (13) we get,

$$\boldsymbol{u_1' = u_2}$$
$$\boldsymbol{u_2' = u_3}$$

$$\left(F\left(u_2 - (1 - e^{-\eta})u_3(0)\right) + K + M - 1\right)\left(u_2 - (1 - e^{-\eta})u_3(0)\right) + ((u_3 - e^{-\eta}u_3(0))^{n-1}\left({u_3}' + e^{-\eta}u_3(0)\right) + \frac{2n}{n+1}\left(u_1 - (\eta - 1 + e^{-\eta})u_3(0)\right)((u_3 - e^{-\eta}u_3(0)) = 0 \tag{14}$$

Now for equation (4) and (5) we seek two shape function for $\theta(\eta)$ satisfying

$$p_1(0) = 1,\quad p_1(\infty) = 0$$
$$p_2(0) = 0,\quad p_2(\infty) = 1$$

Hence, we have

$$p_1(\eta) = e^{-\eta},\ p_2(\eta) = 1 - e^{-\eta} \tag{15}$$

Then we consider the variable transformation from $\theta(\eta)$ to $v(\eta)$ by

$$\theta(\eta) = v(\eta) - s_1(\eta)[v(0) - 1] - s_2(\eta)\, v'(\infty) \tag{16}$$

Hence, $\theta(\eta)$ in Eq. (16) is a boundary shape function, which for any function $v(\eta)$ automatically satisfies the boundary conditions:

$$\theta(0) = 1, \quad \theta(\infty) = 0, \tag{17}$$

Now by substituting the $v(\eta)$ by $\theta(\eta)$, we get

$$-P_r(n+1)(\varepsilon(-(1-e^{\eta})v(\infty) + v(0) - ve^{\eta} - 1) - e^{\eta})\left(\alpha e^{\eta} f' - \beta(-(1-e^{\eta})v(\infty) + v(0) - ve^{\eta} - 1)\right) + P_r\left(2n\left(-v(\infty)e^{\eta} + ve^{\eta} + v'^{e^{\eta}}\right)e^{-\eta} - \left((1-e^{\eta})v(\infty) - v(0) + ve^{\eta} + 1\right)e^{-\eta}\right)fe^{\eta} + \lambda(n+1)(-(1-e^{\eta})v(\infty) + v(0) - ve^{\eta} - 1)f')e^{\eta} - (n+1)(\varepsilon(-(1-e^{\eta})v(\infty) + v(0) - ve^{\eta} - 1) - e^{\eta})(-((e^{\eta} - 1)v(\infty) + v(0) - ve^{\eta} - 1)e^{-\eta} + v(\infty) - v + v")e^{\eta} + (n+1)(E_c M P_r f'^2) + \varepsilon((-v(\infty)e^{\eta} + ve^{\eta} + e^{\eta}v')e^{-\eta} - ((1-e^{-\eta})v(\infty) - v(0) + ve^{\eta} + 1)e^{-\eta})^2)e^{2\eta})e^{-2\eta} = 0 \tag{18}$$

Now take $v(0) = 1$ in equ. (18) and
Now we reduce the given equation (18) in first order take
$v = v_1, \ \ v' = v_2,$ and put in equation (18) we get

$$v_1' = v_2$$

$$P_r(n+1)\left(\varepsilon\left((1-e^{\eta})v_2(0) + v_1 e^{\eta}\right) + e^{\eta}\right)\left(\alpha e^{\eta} f' + \beta\left((1-e^{\eta})v_2(0) + v_1 e^{\eta}\right)\right) + P_r\left(2n(-(1-e^{\eta})v_2(0) - v_2(0)e^{\eta} + v_2 e^{\eta} - 1)f - \lambda(n+1)\left((1-e^{\eta})v_2(0) + v_1 e^{\eta}\right)f'\right)e^{\eta} - (n+1)\left(\varepsilon\left((1-e^{\eta})v_2(0) + v_1 e^{\eta}\right) + e^{\eta}\right)(-(1-e^{\eta})v_2(0) + (-v_2(0) + v_1 - v'_2)e^{\eta} - v_1 e^{\eta}) + (n+1)E_c M P_r f" + \varepsilon(-(1-e^{\eta})v_2(0) - v_2(0)e^{\eta} + v_2 e^{\eta})^2))e^{-2\eta} = 0 \tag{19}$$

Equation (14) and (19) which can be view as initial value problem wit$h\ u_3(0)\ and\ v_2(0)$ unknown initial condition. We can obtain this unknown initial condition to apply any numerical method and get the exact solution of the given equation.

Summary/Conclusion: -

The governing equations take the form of nonlinear ordinary differential equations with boundary conditions. The boundary shape function approach used to reduce the boundary value problem into an initial value problem. It meets the boundary conditions automatically. Any numerical technique will be used to solve the first-order differential equation system that has been obtained. The outcome of current study will be served as a valuable source of knowledge for practical applications and as a supplement to previous research.

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