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Super Harmonic Mean Labelingof Some Path Related Graphs

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Abstract

A graph G with α points and βlines is known as a super harmonic mean graphs if it is possible to value the points$z \in V$ with differentvalues$g(z)$ from $1,2,\dots,\alpha+\beta$ in such a way that when everyline$l = ab$ is valued with $g(l = ab) = \left\lceil \frac{2g(a)g(b)}{g(a)+g(b)} \right\rceil$ or $\left\lfloor \frac{2g(a)g(b)}{g(a)+g(b)} \right\rfloor$ then the line values are distinct. In this case, g is known asthe **Super harmonic mean labeling of *G***. In this paper, we proved we prove that some path related graphs such as the Path union of two cycles C_m, $k-$ Path union of two cycles C_m, Path union of two crowns C_m^* and $k-$ Path union of two crowns C_m^* all are super harmonic mean graph .

Keywords: Graph, Graph Labeling, Harmonic Mean labeling, Super Harmonic Mean Labeling Path, Cycle, Crown.

AMS subject classification: 05C78.

I. INTRODUCTION

We often refer to finite undirected graphs as "graphs" rather than loops or multiple lines.The cycle of length m is representedby C_m, the crown of length m is represented by C_m^* and therefore the path of length$k$ is represented by P_k. We follow Harary [3] for all other standards in terminology and notation.For a thorough analysis of graph labeling, see Gallain [2].S. Somasundaram et al. introduced the concept of harmonic mean labeling of graphs in [6,7,8].S. Meena. et.al investigate a few harmonic mean graph in [4].R. Ponraj and D. Ramya introduced super mean labeling of graphs in [5]. S. Sandhya and C. David Raj introduced Super harmonic labeling in [9]. C. David Raj and C. Jayasekaran investigate a some results on super harmonic mean graphs [1]. We examine the super harmonic mean labeling of some path related graphs in this paper.For the purpose of this investigation, the definitions presented are below helpful.

Definition 1.1: [4,5,6,7,8]
A closed path is known as a **cycle**. A cycle on $m$points is representedby C_m.
Definition 1.2: [4]
The **union of two graphs**$H_1 = (P_1, Q_1), H_2 = (P_2, Q_2)$ is a graph $H = H_1 \cup H_2$ withpoint set $P = P_1 \cup P_2$ and the line set $Q = Q_1 \cup Q_2$.

Definition 1.3: [4]
Let $H_1, H_2, \ldots, H_m$, $m \geq 2$ be m copies of a fixed graph H. The graph Hgot by joiningaline between H_j and H_{j+1} for $j = 1,2,\ldots,m-1$ is known as a **path union of G.**

Definition 1.4: [4]
The **$k -$ path union of two cycles C_m** is the graph got by joining two points from two copies of C_m by a path P_k of length $k-1$.

II. MAIN RESULTS

We investigate thesuper harmonic mean labeling of some path related graphs in this paper.

Theorem: 2.1

The path union of two cycles C_m is a super harmonic mean graph.
Proof:
Let $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$ be the points of two cycles$C_m$in G.
Let $V(G) = \{a_1, a_2, \ldots, a_m, b_1, b_2, \ldots, b_m\}$
$E(G) = \{a_\ell a_{\ell+1} \,/\, 1 \leq \ell \leq m-1\} \cup \{b_\ell b_{\ell+1} \,/\, 1 \leq \ell \leq m-1\}$
$\cup \{a_m a_1, b_m b_1, a_m b_1\}$.

Which are denoted in Figure 1

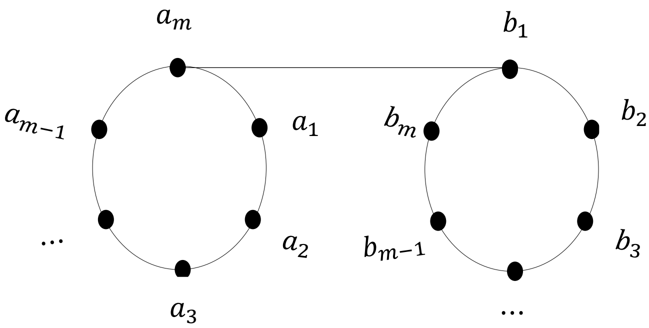

Figure 1: Super harmonic mean labeling of path union of two cycles C_m

A mapping$g: V(G) \to \{1,2,\ldots,4m+1\}$ by
$g(a_\ell) = 2\ell$ for$1 \leq \ell \leq m$
$g(b_\ell) = 2m + \ell$ for$1 \leq \ell \leq m$
The linevalues are different.
Therefore, g is the super harmonic mean labeling of G.

Example: 2.1.1

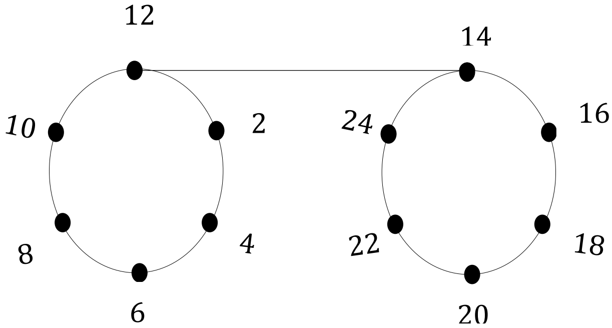

Figure 2: super harmonic mean labeling of path union of two cycles C_6

Theorem: 2.2
$k-$ Path union of two cycles C_m is a super harmonic mean graph.
Proof:
Let $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ be the points of two cycles $C_m$in G.
Let $a_m = c_1, c_2, \dots, c_k = b_1$ be the points of path P_k.
Let $V(G) = \begin{Bmatrix} a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_m, \\ c_1, c_2, \dots, c_k \end{Bmatrix}$
$E(G) = \{a_\ell a_{\ell+1}/1 \le \ell \le m-1\} \cup \{b_\ell b_{\ell+1}/1 \le \ell \le m-1\}$
$\cup \{c_\ell c_{\ell+1}/1 \le \ell \le k-1\} \cup \{a_m a_1, b_m b_1\}$

Which are denoted in Figure 3

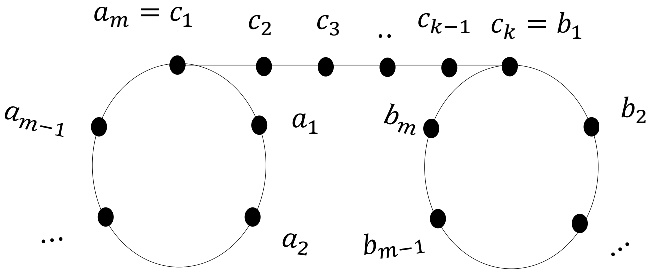

Figure 3: Super harmonic mean labeling of $k-$ path union of two cycles C_m

Amapping$g: V(G) \rightarrow \{1,2, \dots, 4m + k + 3\}$ by
$g(a_\ell) = 2\ell$ for$1 \le \ell \le m$
$g(c_\ell) = 2m + 2\ell - 2$ for$2 \le \ell \le k-1$
$g(b_\ell) = 2m + k + 2\ell + 2$ for$1 \le \ell \le m$
The line values are different.
Therefore, g is the super harmonic mean labelingof G.

Example: 2.2.1

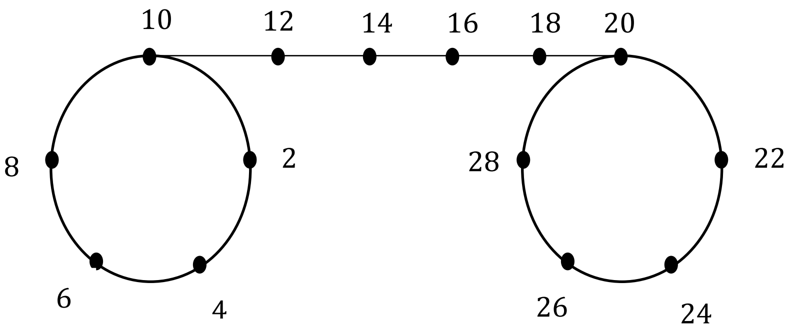

Figure 4: Super harmonic mean labeling of $k-$ path union of two cycles C_5

Theorem: 2.3
Path union of two crowns C_m^* is asuper harmonic mean graph.
Proof:
Let $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ be the points of two cycles $C_m$in G.
Let $a'_1, a'_2, \dots, a'_m$ be the pendant points attached at $a_1, a_2, \dots, a_m$ respectively and $b'_1, b'_2, \dots, b'_m$ be the pendant points attached at $b_1, b_2, \dots, b_m$ respectively.
Let $V(G) = \{a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_m, a'_1, a'_2, \dots, a'_m, b'_1, b'_2, \dots, b'_m\}$
$E(G) = \{a_\ell a_{\ell+1} \ / \ 1 \le \ell \le m-1\} \cup \{b_\ell b_{\ell+1} \ / \ 1 \le \ell \le m-1\}$
$\cup \{a_\ell a'_\ell \ / \ 1 \le \ell \le m\} \cup \{b_\ell b'_\ell \ / \ 1 \le \ell \le m\}$
$\cup \{a_m a_1, b_m b_1, a_m b_1\}$.

Which are denoted in Figure 5

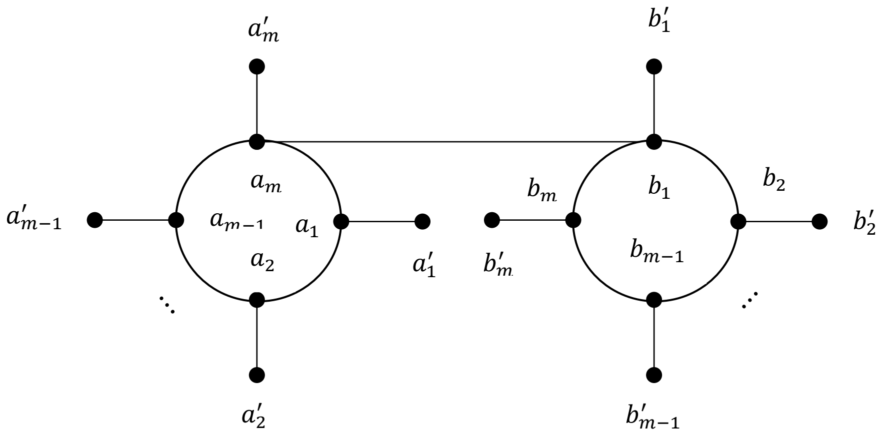

Figure 5: Sper harmonic mean labeling of Path union of two crowns C_m^*

Amapping$g: V(G) \to \{1,2,\dots,8m\}$ by
$g(a_\ell) = 4\ell$ for$1 \le \ell \le m$
$g(a'_\ell) = 4\ell - 2$ for$1 \le \ell \le m$
$g(b_\ell) = 4m + 4\ell$ for$1 \le \ell \le m$
$g(b'_\ell) = 4m + 4\ell - 2$ for$1 \le \ell \le m$
The line values are different.
Therefore, g is the super harmonic mean labelingof G.

Example: 2.3.1

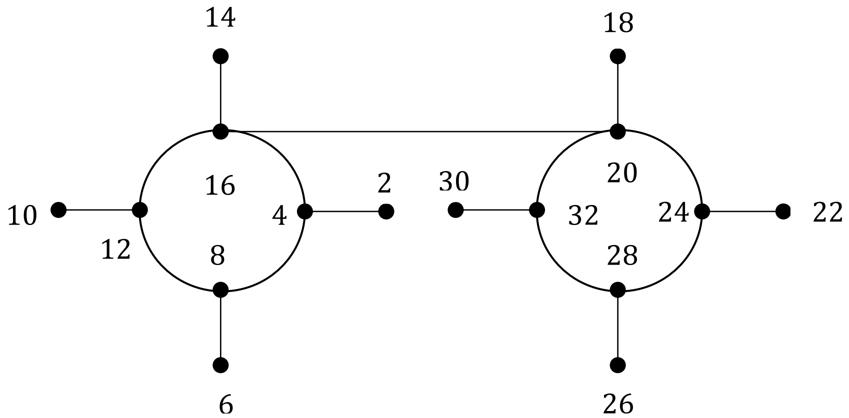

Figure 6: Super harmonic mean labeling of path union of two crowns C_4^*

Theorem: 2.4
$k -$ Path union of two crowns C_m^* is asuper harmonic mean graph.
Proof:
Let $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ be the points of two cycles $C_m$in G.
Let $a_m = c_1, c_2, \dots, c_k = b_1$ be the points of path P_k.
Let $a'_1, a'_2, \dots, a'_m$ be the pendant points attached at $a_1, a_2, \dots, a_m$ respectively and $b'_1, b'_2, \dots, b'_m$ be the pendant points attached at $b_1, b_2, \dots, b_m$ respectively.
Let $V(G) = \begin{Bmatrix} a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_m, c_1, c_2, \dots, c_k, \\ a'_1, a'_2, \dots, a'_m, b'_1, b'_2, \dots, b'_m \end{Bmatrix}$
$E(G) = \{a_\ell a_{\ell+1} / 1 \le \ell \le m-1\} \cup \{b_\ell b_{\ell+1} / 1 \le \ell \le m-1\}$
$\cup \{c_\ell c_{\ell+1} / 1 \le \ell \le k-1\} \cup \{a_\ell a'_\ell \,/\, 1 \le \ell \le m\}$
$\cup \{b_\ell b'_\ell \,/\, 1 \le \ell \le m\} \cup \{a_m a_1, b_m b_1\}$
Which are denoted in Figure 7

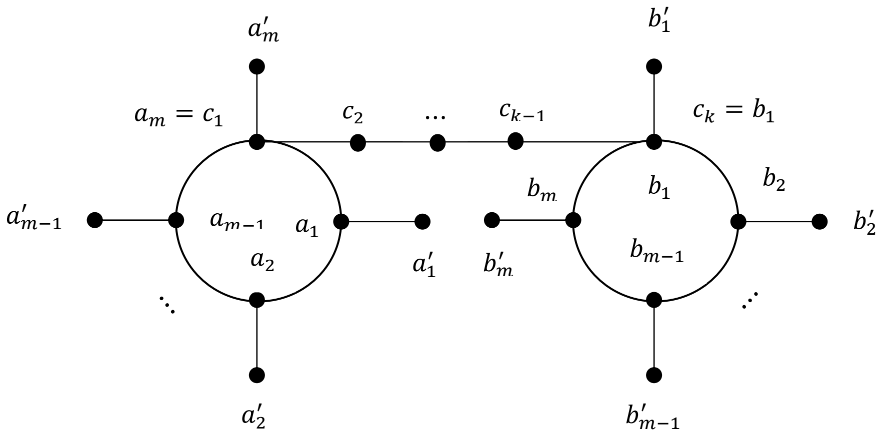

Figure 7: Super harmonic mean labeling of k −path union of two crowns C_m^*

Amapping$g: V(G) \to \{1,2,\ldots,8m+k+2\}$ by
$g(a_\ell) = 4\ell$ for$1 \le \ell \le m$
$g(a'_\ell) = 4\ell - 2$ for$1 \le \ell \le m$
$g(c_\ell) = 4m + 2\ell - 2$ for$2 \le \ell \le k-1$
$g(b_\ell) = 4m + 2k + 4\ell - 4$ for$1 \le \ell \le m$
$g(b'_\ell) = 4m + 2k + 4\ell - 6$ for$1 \le \ell \le m$
The line values are different.
Therefore, g is the super harmonic mean labelingof G.

Example: 2.4.1

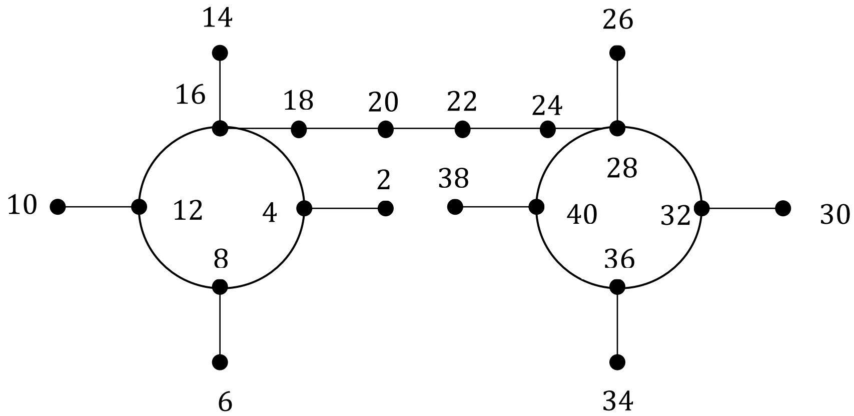

Figure 8: Super harmonic mean labeling of k −path union of two crowns C_4^*

III. CONCLUSION

Four new findings on thesuper harmonic mean labeling of unique graphs like the Path, Cycle, and Crown have been presented.Similar work can be done for more families as well as with various graph labeling methods.

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