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Quotient-4 Cordial Labeling Of Some Caterpillar And Lobster Graphs

S. Kavitha[1*], Dr. P. Sumathi[2]

[1]Department of mathematics, St.Thomas College of Arts and Science, Koyambedu, Chennai-600107, India.*
Email:Kavinu76@gmail.com
[2]Department of mathematics, C.Kandaswami Naidu College for Men, Annanagar, Chennai-600102, India.
Email: sumathipaul@yahoo.co.in

*[*]**Corresponding Author:** S. Kavitha*
[*]Department of mathematics, St.Thomas College of Arts and Science, Koyambedu, Chennai-600107, India.
Email:Kavinu76@gmail.com

Abstract

Let G (V, E) be a simple graph of order p and size q. Let φ: V (G) → Z_5 – {0} be a function. For each edge set E (G) define the labeling φ:E (G) → Z_4 by $\varphi^*(uv)=\left\lceil\frac{\varphi(u)}{\varphi(v)}\right\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1$, $1 \leq i, j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3$, $k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y. Here some caterpillar graphs such as star graph (S_n), Bistar graph ($B_{n,n}$), P_n [N] graph, P_n [N_o] graph, P_n [N_e] graph, Twig graph (T_m), ($P_n \odot K_{1,r}$), S(S_n), S($B_{n,n}$), S(P_n [N]), S(P_n [N_o]), S(P_n [N_e]), S(T_m) and S($P_n \odot K_{1,r}$) graph proved to be quotient-4 cordial graphs.*

***Keywords**: Caterpillar graph, lobster graph, Star graph, Bistar graph, Twig graph, subdivision graph, Quotient-4 cordial labeling, Quotient-4 cordial graph.*

1. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non-trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [6] for more information. The cordial labeling concept was first introduced by Cahit [2]. H- and H2 –cordial labeling was introduced by Freeda S and Chellathurai R.S [4]. Mean Cordial Labeling was introduced by Albert William, Indira Rajasingh, and S Roy [1]. A graph G is said to be quotient-4 cordial graph if it receives quotient-4 cordial labeling .Let $v_\varphi(i)$ denotes the number of vertices labeled with i and $e_\varphi(k)$ denotes the number of edges labeled with k, $1 \leq i \leq 4$, $0 \leq k \leq 3$.

2. DEFINITIONS

Definition: 2.1[9] Let G (V, E) be a simple graph of order p and size q. Letφ: V (G) → Z_5 – {0} be a function. For each edge set E (G) define the labeling φ^*: E (G) → Z_4 by $\varphi^*(uv)=\left\lceil\frac{\varphi(u)}{\varphi(v)}\right\rceil$ (mod 4) where $\varphi(u) \geq \varphi(v)$. The function φ is called Quotient-4 cordial labeling of G if $|v_\varphi(i) - v_\varphi(j)| \leq 1, 1 \leq i,$

$j \leq 4$, $i \neq j$ where $v_\varphi(x)$ denote the number of vertices labeled with x and $|e_\varphi(k) - e_\varphi(l)| \leq 1$, $0 \leq k, l \leq 3$, $k \neq l$, where $e_\varphi(y)$ denote the number of edges labeled with y.

Definition: 2.2[3] A **tree** is a connected acyclic graph.

Definition: 2.3[3] A **caterpillar** is a tree whose non-leaf vertices form a path.

Definition: 2.4[7] A **lobster** is a tree with the property that the removal of the end points leaves a caterpillar.

Definition: 2.5[10] A **star** graph is a complete bipartite graph $K_{1,n}$ and it is denoted by S_n.

Definition: 2.6[10] A **bistar** graph $B_{n,n}$, is the graph obtained by joining the center vertex of two copies of $K_{1,n}$ by an edge.

Definition: 2.7 A graph $P_n[N]$, $n \geq 2$ is obtained from a path of length n-1 by attaching r pendant edges to each r^{th} vertex, $1 \leq r \leq n$ of the path P_n.

Definition: 2.8 A graph $P_n[N_o]$, $n \geq 3$ is obtained from a path of length n-1 by attaching 2r-1 pendant edges to each $(2r-1)^{th}$ vertex, $1 \leq r \leq n$ of the path P_n.

Definition: 2.9 A graph $P_n[N_e]$, $n \geq 2$ is obtained from a path of length n-1 by attaching 2r pendant edges to each $(2r)^{th}$ vertex, $1 \leq r \leq n$ of the path P_n.

Definition: 2.10[8] A graph got from a path by attaching only two pendent edges to all vertices except the end vertices of the path is called a **twig graph** T_m, where m is the set of internal vertices. Generally a twig graph T_m has 3m + 2 vertices and 3m + 1 edges.

Definition: 2.11[5] If x = uv is a line of G and w is not a point of G, then the line x is said to be subdivided when it is replaced by lines uw and wv. If every line of G is subdivided, the resulting graph is the **subdivision graph** S (G).

3. MAIN RESULT

3.1 SOME TYPES OF CATERPILLAR GRAPHS.

Theorem 3.1.1: Any star graph $G = S_n$ is quotient-4 cordial.

Proof: Let $V(G) = \{u, v_i : 1 \leq i \leq n\}$ and $E(G) = \{(uv_i) : 1 \leq i \leq n\}$.

Here $|V(G)| = n + 1$, $|E(G)| = n$.

Define $\varphi : V(G) \to \{1, 2, 3, 4\}$ by $\varphi(u) = 1$.

For i, $1 \leq i \leq n$.

$\varphi(v_i) = 1$, if $i \equiv 0 \pmod 4$.

$\varphi(v_i) = 2$, if $i \equiv 3 \pmod 4$.

$\varphi(v_i) = 3$, if $i \equiv 2 \pmod 4$.

$\varphi(v_i) = 4$, if $i \equiv 1 \pmod 4$.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0 \pmod 4$	$\frac{n}{4}+1$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$
$n \equiv 1 \pmod 4$	$\frac{n+3}{4}$	$\frac{n+3}{4}-1$	$\frac{n+3}{4}-1$	$\frac{n+3}{4}$
$n \equiv 2 \pmod 4$	$\frac{n+2}{4}$	$\frac{n+2}{4}-1$	$\frac{n+2}{4}$	$\frac{n+2}{4}$
$n \equiv 3 \pmod 4$	$\frac{n+1}{4}$	$\frac{n+1}{4}$	$\frac{n+1}{4}$	$\frac{n+1}{4}$

Table 3.1.1

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0 \pmod 4$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$	$\frac{n}{4}$
$n \equiv 1 \pmod 4$	$\frac{n+3}{4}$	$\frac{n+3}{4}-1$	$\frac{n+3}{4}-1$	$\frac{n+3}{4}-1$
$n \equiv 2 \pmod 4$	$\frac{n+2}{4}$	$\frac{n+2}{4}-1$	$\frac{n+2}{4}-1$	$\frac{n+2}{4}$
$n \equiv 3 \pmod 4$	$\frac{n+1}{4}$	$\frac{n+1}{4}-1$	$\frac{n+1}{4}$	$\frac{n+1}{4}$

Table 3.1.2

The above tables 3.1.1 and 3.1.2 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the star graph S_n is quotient-4 cordial labeling.

Theorem: 3.1.2 The Bistar graph G = $B_{n,n}$ is quotient-4 cordial.
Proof: Let V (G) = {u, v, u_i, v_i: $1 \le i \le n$ } and E (G) = {(uv), (uu_i), (vv_i) : $1 \le i \le n$}.
Here $|V(G)|$ = 2n + 2, $|E(G)|$ = 2n + 1.
Define φ : V (G) → {1, 2, 3, 4}.
When n = 1, φ (u) = 4, φ (v) =1, φ (u_1) = 2 and φ (v_1) = 3.
When n > 1, Assign φ (u) = φ (v) = 1 and φ (u_1) = 4, φ (u_2) = 3.
Labeling of the vertices u_i, for i, $3 \le i \le n$ is given below.
φ (u_i) = 3, if i ≡ 1 (mod 2).
φ (u_i) = 1, if i ≡ 0 (mod 2).
Labeling of the vertices v_i, For i, $1 \le i \le n$ is given below.
φ (v_i) = 2, if i ≡ 1 (mod 2).
φ (v_i) = 4, if i ≡ 0 (mod 2).

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
n ≡ 0(mod 4)	$\frac{2n}{4}+1$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}+1$
n ≡ 1(mod 4)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$
n ≡ 2(mod 4)	$\frac{2n}{4}+1$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}+1$
n ≡ 3(mod 4)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$

Table 3.1.3

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
n ≡ 0(mod 4)	$\frac{2n}{4}+1$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}$
n ≡ 1(mod 4)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}-1$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$
n ≡ 2(mod 4)	$\frac{2n}{4}+1$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}$
n ≡ 3(mod 4)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}-1$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$

Table 3.1.4

The above tables 3.1.3 and 3.1.4 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the Bistar graph $B_{n,n}$ is quotient-4 cordial labeling.

Theorem: 3.1.3 A graph P_n[N] is quotient-4 cordial if n ≥ 2.
Proof: Let G be a P_n[N] graph. Let V (G) = {u_α : $1 \le \alpha \le$ n} ∪ {$v_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1 \le \beta \le \alpha$} and E (G) = {$u_\alpha u_{\alpha+1}$: $1 \le \alpha \le n$-1} ∪ { $u_\alpha v_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1 \le \beta \le \alpha$}.
Here $|V(G)| = \frac{n^2+3n}{2}$, $|E(G)| = \frac{n^2+3n-2}{2}$.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's are given below.
For $1 \le \alpha \le$ n.
φ (u_α) = 1.
Labeling of $v_{\alpha,\beta}$'s are given below.
For $1 \le \alpha \le$ n, $1 \le \beta \le \alpha$.
When $\alpha \equiv 0, 7$ (mod 8).
φ ($v_{\alpha,\beta}$) = 1 if β ≡ 1 (mod 4) and β ≠ 1.
φ ($v_{\alpha,\beta}$) = 2 if β ≡ 2 (mod 4).
φ ($v_{\alpha,\beta}$) = 3 if β ≡ 3 (mod 4).
φ ($v_{\alpha,\beta}$) = 4 if β ≡ 0 (mod 4) and β = 1.

When $\alpha \equiv 1, 6$ (mod 8).
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 0$ (mod 4).
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 1$ (mod 4).
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 2$ (mod 4).
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 3$ (mod 4).
When $\alpha \equiv 2, 5$ (mod 8).
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 2$ (mod 4) and $\beta \neq 2$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 3$ (mod 4).
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 0$ (mod 4) and $\beta = 2$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 1$ (mod 4).
When $\alpha \equiv 3, 4$ (mod 8).
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 3$ (mod 4) and $\beta \neq 3$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 0$ (mod 4) and $\beta = 3$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 1$ (mod 4).
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 2$ (mod 4).

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0, 5$(modulo 8)	$\frac{n^2+3n}{8}$	$\frac{n^2+3n}{8}$	$\frac{n^2+3n}{8}$	$\frac{n^2+3n}{8}$
$n \equiv 1,4$(modulo 8)	$\frac{n^2+3n+4}{8}$	$\frac{n^2+3n+4}{8}$	$\frac{n^2+3n+4}{8}-1$	$\frac{n^2+3n+4}{8}-1$
$n \equiv 2, 3$(modulo 8)	$\frac{n^2+3n-2}{8}+1$	$\frac{n^2+3n-2}{8}$	$\frac{n^2+3n-2}{8}$	$\frac{n^2+3n-2}{8}$
$n \equiv 6, 7$(modulo 8)	$\frac{n^2+3n+2}{8}$	$\frac{n^2+3n+2}{8}$	$\frac{n^2+3n+2}{8}$	$\frac{n^2+3n+2}{8}-1$

Table 3.1.5

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0, 5$(modulo 8)	$\frac{n^2+3n}{8}$	$\frac{n^2+3n}{8}-1$	$\frac{n^2+3n}{8}$	$\frac{n^2+3n}{8}$
$n \equiv 1,4$(modulo 8)	$\frac{n^2+3n-4}{8}$	$\frac{n^2+3n-4}{8}$	$\frac{n^2+3n-4}{8}+1$	$\frac{n^2+3n-4}{8}$
$n \equiv 2, 3$(modulo 8)	$\frac{n^2+3n-2}{8}$	$\frac{n^2+3n-2}{8}$	$\frac{n^2+3n-2}{8}$	$\frac{n^2+3n-2}{8}$
$n \equiv 6, 7$(modulo 8)	$\frac{n^2+3n+2}{8}-1$	$\frac{n^2+3n+2}{8}-1$	$\frac{n^2+3n+2}{8}$	$\frac{n^2+3n+2}{8}$

Table 3.1.6
The above tables 3.1.5 and 3.1.6 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph $P_n[N]$ is quotient-4 cordial labeling.

Theorem: 3.1.4 A graph $P_n[N_o]$ is quotient-4 cordial if n is odd and $n \geq 3$.
Proof: Let G be a $P_n[N_o]$ graph. Let V (G) = $\{u_\alpha : 1 \leq \alpha \leq n\} \cup \{v_{\alpha,\beta} : 1 \leq \alpha \leq n, \alpha \equiv 1 \text{ (mod 2)}, 1 \leq \beta \leq \alpha\}$ and E (G) = $\{u_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n\text{-}1\} \cup \{u_\alpha v_{\alpha,\beta} : 1 \leq \alpha \leq n, \alpha \equiv 1 \text{ (mod 2)}, 1 \leq \beta \leq \alpha\}$.
Here $|V(G)| = \frac{n^2+6n+1}{4}$, $|E(G)| = \frac{n^2+6n-3}{4}$.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's are given below.
Case 1:
For $n \leq 7$.
$\varphi(u_1) = 3$.
$\varphi(u_\alpha) = 1$ if $2 \leq \alpha \leq n$.
Case 2:
For $n \geq 9$.
$\varphi(u_\alpha) = 1$ if $1 \leq \alpha \leq n$.
Labeling of $v_{\alpha,\beta}$'s are given below.
For $1 \leq \alpha \leq n$.
For α = 1, 7.

$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 3$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 3$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 0 \pmod 3$.
For $\alpha = 3, 5, 9$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 0 \pmod 3$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 1 \pmod 3$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 2 \pmod 3$.
For $\alpha \equiv 1 \pmod 4$ and $\alpha \geq 13$.
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 0 \pmod 4$ and $\beta \neq 4$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 3 \pmod 4$ and $\beta = 4$.
For $\alpha \equiv 3 \pmod 4$ and $\alpha \geq 11$.
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 1 \pmod 4$ and $\beta \neq 1, 5$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 2 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 3 \pmod 4$ and $\beta = 1$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 0 \pmod 4$ and $\beta = 5$.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n = 5$	$\frac{n^2+6n-7}{16}+1$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}+1$	$\frac{n^2+6n-7}{16}$
$n \equiv 1$(modulo 4) and $n \neq 5$	$\frac{n^2+6n-7}{16}+1$	$\frac{n^2+6n-7}{16}+1$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}$
$n \equiv 3$(modulo 4)	$\frac{n^2+6n+5}{16}$	$\frac{n^2+6n+5}{16}$	$\frac{n^2+6n+5}{16}$	$\frac{n^2+6n+5}{16}-1$

Table 3.1.7

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n = 5$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}+1$
$n \equiv 1$(modulo 4) and $n \neq 5$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}$	$\frac{n^2+6n-7}{16}+1$	$\frac{n^2+6n-7}{16}$
$n \equiv 3$(modulo 4)	$\frac{n^2+6n+5}{16}-1$	$\frac{n^2+6n+5}{16}-1$	$\frac{n^2+6n+5}{16}$	$\frac{n^2+6n+5}{16}$

Table 3.1.8

The above tables 3.1.7 and 3.1.8 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$.
Hence the graph $P_n[N_o]$ is quotient-4 cordial labeling.

Theorem: 3.1.5 A graph $P_n[N_e]$ is quotient-4 cordial if n is even and $n \geq 2$.
Proof: Let G be a $P_n[N_e]$ graph. Let $V(G) = \{u_\alpha : 1 \leq \alpha \leq n\} \cup \{v_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\}$ and $E(G) = \{u_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n-1\} \cup \{u_\alpha v_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\}$.
Here $|V(G)| = \frac{n^2+6n}{4}$, $|E(G)| = \frac{n^2+6n-4}{4}$.
Define $\varphi: V(G) \to \{1, 2, 3, 4\}$.
Labeling of u_α's are given below.
For $1 \leq \alpha \leq n$.
$\varphi(u_1) = 4$.
$\varphi(u_\alpha) = 1$ if $2 \leq \alpha \leq n$.
Labeling of $v_{\alpha,\beta}$'s are given below.
For $2 \leq \alpha \leq n$.
When $\alpha \equiv 0, 6 \pmod 8$.
$\varphi(v_{\alpha,\beta}) = 1$ if $\beta \equiv 2 \pmod 4$ and $\beta \neq 2, 6$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 3 \pmod 4$ and $\beta = 2$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 0 \pmod 4$ and $\beta = 6$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 1 \pmod 4$.

When $\alpha \equiv 2, 4$ (mod 8).
φ ($v_{\alpha,\beta}$) = 1 if β ≡ 0 (mod 4) and β ≠ 4.
φ ($v_{\alpha,\beta}$) = 2 if β ≡ 1 (mod 4).
φ ($v_{\alpha,\beta}$) = 3 if β ≡ 2 (mod 4).
φ ($v_{\alpha,\beta}$) = 4 if β ≡ 3 (mod 4) and β = 4.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0{,}2$(modulo 8)	$\frac{n^2+6n}{16}$	$\frac{n^2+6n}{16}$	$\frac{n^2+6n}{16}$	$\frac{n^2+6n}{16}$
$n \equiv 4{,}6$(modulo 8)	$\frac{n^2+6n-8}{16}+1$	$\frac{n^2+6n-8}{16}$	$\frac{n^2+6n-8}{16}$	$\frac{n^2+6n-8}{16}+1$

Table 3.1.9

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0{,}2$(modulo 8)	$\frac{n^2+6n}{16}$	$\frac{n^2+6n}{16}-1$	$\frac{n^2+6n}{16}$	$\frac{n^2+6n}{16}$
$n \equiv 4{,}6$(modulo 8)	$\frac{n^2+6n-8}{16}+1$	$\frac{n^2+6n-8}{16}$	$\frac{n^2+6n-8}{16}$	$\frac{n^2+6n-8}{16}$

Table 3.1.10
The above tables 3.1.9 and 3.1.10 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$.
Hence the graph $P_n [N_e]$ is quotient-4 cordial labeling.

Theorem: 3.1.6 A Twig graph T_m is quotient-4 cordial if m ≥ 3.
Proof: Let G be a T_m graph. Let V (G) = $\{u_\alpha : 1 \leq \alpha \leq m\} \cup \{v_\alpha : 1 \leq \alpha \leq 2m-4\}$.
E (G) = $\{u_\alpha u_{\alpha+1} : 1 \leq \alpha \leq m-1\} \cup \{ u_\alpha v_\beta : 2 \leq \alpha \leq m-1, 2\alpha - 3 \leq \beta \leq 2\alpha - 2 \}$.
Here $|V(G)|$= 3m – 4, $|E(G)|$ = 3m – 5.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's are given below.
For $1 \leq \alpha \leq m$.
φ (u_α) = 1 if $\alpha \equiv 1, 2, 3$ (mod 4) and $\alpha \neq 1$.
φ (u_α) = 2 if $\alpha = 1$.
φ (u_α) = 3 if $\alpha \equiv 0$ (mod 4).
Labeling of v_β's are given below.
For $1 \leq \beta \leq 2m-4$.
φ (v_β) = 2 if $\beta \equiv 0, 4, 6$ (mod 8).
φ (v_β) = 3 if $\beta \equiv 2, 5$ (mod 8).
φ (v_β) = 4 if $\beta \equiv 1, 3, 7$ (mod 8).

Nature of m	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
m≡ 0(modulo 4)	$\frac{3m-4}{4}$	$\frac{3m-4}{4}$	$\frac{3m-4}{4}$	$\frac{3m-4}{4}$
m≡ 1(modulo 4)	$\frac{3m-3}{4}$	$\frac{3m-3}{4}$	$\frac{3m-3}{4}$	$\frac{3m-3}{4}-1$
m≡ 2(modulo 4)	$\frac{3m-6}{4}+1$	$\frac{3m-6}{4}+1$	$\frac{3m-6}{4}$	$\frac{3m-6}{4}$
m≡ 3(modulo 4)	$\frac{3m-5}{4}+1$	$\frac{3m-5}{4}$	$\frac{3m-5}{4}$	$\frac{3m-5}{4}$

Table 3.1.11

Nature of m	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
m≡ 0(modulo 4)	$\frac{3m-4}{4}$	$\frac{3m-4}{4}-1$	$\frac{3m-4}{4}$	$\frac{3m-4}{4}$
m≡ 1(modulo 4)	$\frac{3m-3}{4}-1$	$\frac{3m-3}{4}-1$	$\frac{3m-3}{4}$	$\frac{3m-3}{4}$
m≡ 2(modulo 4)	$\frac{3m-6}{4}$	$\frac{3m-6}{4}$	$\frac{3m-6}{4}+1$	$\frac{3m-6}{4}$
m≡ 3(modulo 4)	$\frac{3m-5}{4}$	$\frac{3m-5}{4}$	$\frac{3m-5}{4}$	$\frac{3m-5}{4}$

Table 3.1.12

The above tables 3.1.11 and 3.1.12 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$.
Hence the graph T_m is quotient-4 cordial labeling.

Theorem: 3.1.7 A graph ($P_n \odot K_{1, r}$) is quotient-4 cordial if $r \geq 3$.
Proof: Let G be a ($P_n \odot K_{1, r}$) graph. Let $V(G) = \{x_\alpha : 1 \leq \alpha \leq n\} \cup \{y_{\alpha,\beta} : 1 \leq \alpha \leq n, 1\leq \beta \leq r\}$ and $E(G) = \{x_\alpha x_{\alpha+1} : 1 \leq \alpha \leq n\text{-}1\} \cup \{x_\alpha y_{\alpha,\beta} : 1 \leq \alpha \leq n, 1\leq \beta \leq r\}$.
Here $|V(G)| = n(r+1)$, $|E(G)| = n(r+1) - 1$.
Define $\varphi: V(G) \to \{1, 2, 3, 4\}$.
Labeling of x_α's are given below.
For $1 \leq \alpha \leq n$.
$\varphi(x_\alpha) = 1$.
Labeling of $y_{\alpha,\beta}$'s are given below.
Case 1:
When $r \equiv 0 \pmod 4$.
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 1$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 3 \pmod 4$.
For $\alpha \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,r}) = 4$.
For $\alpha \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,r}) = 1$.
For $\alpha \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,r}) = 2$.
For $\alpha \equiv 3 \pmod 4$.
$\varphi(y_{\alpha,r}) = 3$.
Case 2:
When $r \equiv 1 \pmod 4$.
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 2$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 3 \pmod 4$.
For $\alpha \equiv 0 \pmod 2$.
$\varphi(y_{\alpha,r}) = 4$, $\varphi(y_{\alpha,r-1}) = 3$.
For $\alpha \equiv 1 \pmod 2$.
$\varphi(y_{\alpha,r}) = 2$, $\varphi(y_{\alpha,r-1}) = 1$.
Case 3:
When $r \equiv 2 \pmod 4$.
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 3$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 3 \pmod 4$.
For $\alpha \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,r}) = 4$, $\varphi(y_{\alpha,r-1}) = 3$, $\varphi(y_{\alpha,r-2}) = 2$.
For $\alpha \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,r}) = 3$, $\varphi(y_{\alpha,r-1}) = 2$, $\varphi(y_{\alpha,r-2}) = 1$.
For $\alpha \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,r}) = 4$, $\varphi(y_{\alpha,r-1}) = 2$, $\varphi(y_{\alpha,r-2}) = 1$.
For $\alpha \equiv 3 \pmod 4$.
$\varphi(y_{\alpha,r}) = 4$, $\varphi(y_{\alpha,r-1}) = 3$, $\varphi(y_{\alpha,r-2}) = 1$.
Case 4:
When $r \equiv 3 \pmod 4$.
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r$.

$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 3 \pmod 4$.

Nature of r and n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$r \equiv 0,2$(modulo 4) $n \equiv 0$(modulo 4)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$
$r \equiv 0$(modulo 4) $n \equiv 1$(modulo 4)	$\frac{n(r+1)-1}{4}+1$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$
$r \equiv 0,2$(modulo 4) $n \equiv 2$(modulo 4)	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}$
$r \equiv 0$(modulo 4) $n \equiv 3$(modulo 4)	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}-1$
$r \equiv 1$(modulo 4) $n \equiv 0$(modulo 2)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$
$r \equiv 1$(modulo 4) $n \equiv 1$(modulo 2)	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}$
$r \equiv 2$(modulo 4) $n \equiv 1$(modulo 4)	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}-1$
$r \equiv 2$(modulo 4) $n \equiv 3$(modulo 4)	$\frac{n(r+1)-1}{4}+1$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$
$r \equiv 3$(modulo 4) $n \equiv 0,1$(modulo 2)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$

Table 3.1.13

Nature of r and n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$r \equiv 0,2$(modulo 4) $n \equiv 0$(modulo 4)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}-1$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$
$r \equiv 0$(modulo 4) $n \equiv 1$(modulo 4)	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$
$r \equiv 0,2$(modulo 4) $n \equiv 2$(modulo 4)	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}$
$r \equiv 0$(modulo 4) $n \equiv 3$(modulo 4)	$\frac{n(r+1)+1}{4}-1$	$\frac{n(r+1)+1}{4}-1$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$
$r \equiv 1$(modulo 4) $n \equiv 0$(modulo 2)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}-1$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$
$r \equiv 1$(modulo 4) $n \equiv 1$(modulo 2)	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}$	$\frac{n(r+1)-2}{4}+1$	$\frac{n(r+1)-2}{4}$
$r \equiv 2$(modulo 4) $n \equiv 1$(modulo 4)	$\frac{n(r+1)+1}{4}-1$	$\frac{n(r+1)+1}{4}-1$	$\frac{n(r+1)+1}{4}$	$\frac{n(r+1)+1}{4}$
$r \equiv 2$(modulo 4) $n \equiv 3$(modulo 4)	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$	$\frac{n(r+1)-1}{4}$
$r \equiv 3$(modulo 4) $n \equiv 0,1$(modulo 2)	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}-1$	$\frac{n(r+1)}{4}$	$\frac{n(r+1)}{4}$

Table 3.1.14

The above tables 3.1.13 and 3.1.14 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph ($P_n \odot K_{1,r}$) is quotient-4 cordial labeling.

3.2 SOME TYPES OF LOBSTER GRAPHS.

Theorem: 3.2.1 The graph G = S (S_n) is quotient-4 cordial.

Proof: Let V (G) = {u, u_i ,v_i : $1 \leq i \leq n$ } and E (G) = {(uu_i), (u_iv_i) : $1\leq i \leq n$}.

Here $|V(G)|$ = 2n+1, $|E(G)|$ = 2n.

Define φ : V (G) → {1, 2, 3, 4} by φ (u) = 1.

Labeling of the vertices u_i and v_i.

For i, $1 \leq i \leq n$ is given below.

φ (u_i) = 1, if i ≡ 0 (mod 2).
φ (u_i) = 4, if i ≡ 1 (mod 2).
φ (v_i) = 2, if i ≡ 1 (mod 2).
φ (v_i) = 3, if i ≡ 0 (mod 2) .

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
n ≡ 0(mod 2)	$\frac{2n}{4}+1$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}$
n ≡ 1(mod 2)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}-1$	$\frac{2n+2}{4}$

Table 3.2.1

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
n ≡ 0(mod 2)	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}$	$\frac{2n}{4}$
n ≡ 1(mod 2)	$\frac{2n+2}{4}$	$\frac{2n+2}{4}-1$	$\frac{2n+2}{4}$	$\frac{2n+2}{4}-1$

Table 3.2.2

The above tables 3.2.1 and 3.2.2 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the subdivision of the star graph S (S_n) is quotient-4 cordial labeling.

Theorem: 3.2.2 The graph G = S ($B_{n,n}$) is quotient-4 cordial.
Proof: Let V (G) = {u, v, w, u_i ,v_i ,x_i ,y_i: $1 \le i \le n$ } and E(G) = {(uv), (vw), (uu_i), (u_iv_i), (wx_i), (x_iy_i) : $1\le i \le n$}.
Here $|V(G)|$= 4n+3, $|E(G)|$= 4n+2.
Define φ : V (G) → {1, 2, 3, 4}.
When n = 1, φ (u) = φ (v) = 1, φ (w) = φ (x_1) = 4, φ (v_1) = φ (y_1) = 2, φ (u_1) = 3.
When n > 1, Assign φ (u) = φ (v) = φ (w) = 1, φ (x_1) = φ (x_2) = 4 and φ (y_1) = 2.
Labeling of the vertices u_i and v_i.
For i, $1 \le i \le n$ is given below.
φ (u_i) = 3, φ (v_i) = 2.
Labeling of x_i and y_i is given below.
φ (x_i) = 1, if $3 \le i \le n$.
φ (y_i) = 4, if $2 \le i \le n$.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
n ≡ 0,1(mod 2)	n+1	n+1	n	n+1

Table 3.2.3

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
n ≡ 0,1(mod 2)	n	n+1	n+1	n

Table 3.2.4

The above tables 3.2.3 and 3.2.4 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the subdivision of the Bistar graph S ($B_{n,n}$) is quotient-4 cordial labeling.

Theorem: 3.2.3 A graph S (P_n [N]) is quotient-4 cordial if n ≥ 2.
Proof: Let G be a S (P_n [N]) graph. Let V (G) = {u_α : $1 \le \alpha \le$ n} ∪ {v_α : $1 \le \alpha \le$ n-1} ∪ {$x_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1\le \beta \le \alpha$} ∪ {$y_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1\le \beta \le \alpha$}. E (G) = {$u_\alpha v_\alpha$: $1 \le \alpha \le n-1$} ∪ { $v_\alpha u_{\alpha+1}$: $1 \le \alpha \le$ n-1} ∪ { $u_\alpha x_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1 \le \beta \le \alpha$} ∪ { $x_{\alpha,\beta} y_{\alpha,\beta}$: $1 \le \alpha \le$ n, $1\le \beta \le \alpha$}.
Here $|V(G)|$= $n^2 + 3n - 1$, $|E(G)| = n^2 + 3n - 2$.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's and v_α's are given below.
φ (u_α) = 1 if $1 \le \alpha \le n$.

φ (v_α) = 4 if $1 \le \alpha \le n-1$.
Labeling of $x_{\alpha,\beta}$'s are given below.
When $\alpha = 1, 3$.
φ ($x_{\alpha,\beta}$) = 1 if β = 1.
φ ($x_{\alpha,\beta}$) = 2 if β = 2.
φ ($x_{\alpha,\beta}$) = 3 if β = 3.
When $\alpha = 2$.
φ ($x_{\alpha,\beta}$) = 2 if β = 1.
φ ($x_{\alpha,\beta}$) = 3 if β = 2.
When $4 \le \alpha \le n$.
φ ($x_{\alpha,\beta}$) = 1 if $\beta \equiv 1, 2$ (mod 4) and β ≠ 1, 2.
φ ($x_{\alpha,\beta}$) = 2 if β = 2, 4.
φ ($x_{\alpha,\beta}$) = 3 if β = 1.
φ ($x_{\alpha,\beta}$) = 4 if $\beta \equiv 0, 3$ (mod 4) and β ≠ 4.
Labeling of $y_{\alpha,\beta}$'s are given below.
When $\alpha = 1, 3$.
φ ($y_{\alpha,\beta}$) = 3 if β = 1.
φ ($y_{\alpha,\beta}$) = 2 if β = 2.
φ ($y_{\alpha,\beta}$) = 4 if β = 3.
When $\alpha = 2$.
φ ($y_{\alpha,\beta}$) = 2 if β = 1.
φ ($y_{\alpha,\beta}$) = 4 if β = 2.
When $4 \le \alpha \le n$.
φ ($y_{\alpha,\beta}$) = 1 if β = 1.
φ ($y_{\alpha,\beta}$) = 2 if $\beta \equiv 0, 3$ (mod 4), β = 2 and β ≠ 3, 4.
φ ($y_{\alpha,\beta}$) = 3 if $\beta \equiv 1, 2$ (mod 4), β = 4 and β ≠ 1, 2.
φ ($y_{\alpha,\beta}$) = 4 if β = 3.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0,1$(modulo 4)	$\frac{n^2+3n}{4}$	$\frac{n^2+3n}{4}$	$\frac{n^2+3n}{4} - 1$	$\frac{n^2+3n}{4}$
$n \equiv 2,3$(modulo 4)	$\frac{n^2+3n-2}{4} + 1$	$\frac{n^2+3n-2}{4}$	$\frac{n^2+3n-2}{4}$	$\frac{n^2+3n-2}{4}$

Table 3.2.5

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0,1$(modulo 4)	$\frac{n^2+3n}{4}$	$\frac{n^2+3n}{4} - 1$	$\frac{n^2+3n}{4}$	$\frac{n^2+3n}{4} - 1$
$n \equiv 2,3$(modulo 4)	$\frac{n^2+3n-2}{4}$	$\frac{n^2+3n-2}{4}$	$\frac{n^2+3n-2}{4}$	$\frac{n^2+3n-2}{4}$

Table 3.2.6
The above tables 3.2.5 and 3.2.6 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the graph S ($P_n$ [N]) is quotient-4 cordial labeling.

Theorem: 3.2.4 A graph S (P_n [N_o]) is quotient-4 cordial if n is odd and n ≥ 3.
Proof: Let G be a S (P_n [N_o]) graph. Let V (G) = {u_α : $1 \le \alpha \le$ n} ∪ {v_α : $1 \le \alpha \le$ n-1} ∪ {$x_{\alpha,\beta}$: $1 \le \alpha \le$ n, $\alpha \equiv 1$ (mod 2), 1≤ β ≤ α} ∪ {$y_{\alpha,\beta}$: $1 \le \alpha \le$ n, $\alpha \equiv 1$ (mod 2), 1≤ β ≤ α} and E (G) = {$u_\alpha v_\alpha$: $1 \le \alpha \le n-1$} ∪ { $v_\alpha u_{\alpha+1}$: $1 \le \alpha \le$ n-1} ∪ { $u_\alpha x_{\alpha,\beta}$: $1 \le \alpha \le$ n, $\alpha \equiv 1$ (mod 2), 1 ≤ β ≤ α} ∪ { $x_{\alpha,\beta}\, y_{\alpha,\beta}$: $1 \le \alpha \le$ n, $\alpha \equiv 1$ (mod 2), 1≤ β ≤ α}.
Here $|V(G)| = \frac{n^2+6n-1}{2}$, $|E(G)| = \frac{n^2+6n-3}{2}$.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's and v_α's are given below.
φ (u_α) = 1 if $1 \le \alpha \le n$.
For $1 \le \alpha \le n-1$.
φ (v_α) = 4 if $\alpha \equiv 1$ (mod 2).

$\varphi(v_\alpha) = 3$ if $\alpha \equiv 0 \pmod 2$.

Labeling of $x_{\alpha,\beta}$'s are given below.

When $\alpha = 1$.

$\varphi(x_{\alpha,\beta}) = 3$ if $\beta = 1$.

For $3 \leq \alpha \leq n$ and $\alpha \equiv 1 \pmod 2$.

$\varphi(x_{\alpha,\beta}) = 1$ if $\beta \equiv 2, 3 \pmod 4$, $\beta = 1$ and $\beta \neq 2, 3$.

$\varphi(x_{\alpha,\beta}) = 2$ if $\beta = 2, 3$.

$\varphi(x_{\alpha,\beta}) = 3$ if $\beta = 4$.

$\varphi(x_{\alpha,\beta}) = 4$ if $\beta \equiv 0, 1 \pmod 4$ and $\beta \neq 1, 4$.

Labeling of $y_{\alpha,\beta}$'s are given below.

When $\alpha = 1$.

$\varphi(y_{\alpha,\beta}) = 3$ if $\beta = 1$.

When $3 \leq \alpha \leq n$ and $\alpha \equiv 1 \pmod 2$.

$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 0, 1 \pmod 4$, $\beta = 2$ and $\beta \neq 1, 4$.

$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2, 3 \pmod 4$, $\beta = 4$ and $\beta \neq 2, 3$.

$\varphi(y_{\alpha,\beta}) = 4$ if $\beta = 1, 3$.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 1$(modulo 4)	$\frac{n^2+6n+1}{8}$	$\frac{n^2+6n+1}{8}$	$\frac{n^2+6n+1}{8} - 1$	$\frac{n^2+6n+1}{8}$
$n \equiv 3$(modulo 4)	$\frac{n^2+6n-3}{8} + 1$	$\frac{n^2+6n-3}{8}$	$\frac{n^2+6n-3}{8}$	$\frac{n^2+6n-3}{8}$

Table 3.2.7

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 1$(modulo 4)	$\frac{n^2+6n+1}{8}$	$\frac{n^2+6n+1}{8} - 1$	$\frac{n^2+6n+1}{8}$	$\frac{n^2+6n+1}{8} - 1$
$n \equiv 3$(modulo 4)	$\frac{n^2+6n-3}{8}$	$\frac{n^2+6n-3}{8}$	$\frac{n^2+6n-3}{8}$	$\frac{n^2+6n-3}{8}$

Table 3.2.8

The above tables 3.2.7 and 3.2.8 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph $S(P_n[N_o])$ is quotient-4 cordial labeling.

Theorem: 3.2.5 A graph $S(P_n[N_e])$ is quotient-4 cordial if n is even and $n \geq 2$.

Proof: Let G be a $S(P_n[N_e])$ graph. Let $V(G) = \{u_\alpha : 1 \leq \alpha \leq n\} \cup \{v_\alpha : 1 \leq \alpha \leq n-1\} \cup \{x_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\} \cup \{y_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\}$ and $E(G) = \{u_\alpha v_\alpha : 1 \leq \alpha \leq n-1\} \cup \{v_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n-1\} \cup \{u_\alpha x_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\} \cup \{x_{\alpha,\beta} y_{\alpha,\beta} : 2 \leq \alpha \leq n, \alpha \equiv 0 \pmod 2, 1 \leq \beta \leq \alpha\}$.

Here $|V(G)| = \frac{n^2+6n-2}{2}$, $|E(G)| = \frac{n^2+6n-4}{2}$.

Define $\varphi: V(G) \to \{1, 2, 3, 4\}$.

Labeling of u_α's and v_α's are given below.

$\varphi(u_\alpha) = 1$ if $1 \leq \alpha \leq n$.

For $1 \leq \alpha \leq n-1$.

$\varphi(v_\alpha) = 3$ if $\alpha \equiv 1 \pmod 2$.

$\varphi(v_\alpha) = 4$ if $\alpha \equiv 0 \pmod 2$.

Labeling of $x_{\alpha,\beta}$'s are given below.

For $2 \leq \alpha \leq n$ and $\alpha \equiv 0 \pmod 2$.

$\varphi(x_{\alpha,\beta}) = 1$ if $\beta \equiv 1, 3 \pmod 4$ and $\beta \neq 1, 3$.

$\varphi(x_{\alpha,\beta}) = 2$ if $\beta = 2, 3$.

$\varphi(x_{\alpha,\beta}) = 3$ if $\beta \equiv 0, 2 \pmod 4$ and $\beta \neq 2$.

$\varphi(x_{\alpha,\beta}) = 4$ if $\beta = 1$.

Labeling of $y_{\alpha,\beta}$'s are given below.

When $2 \leq \alpha \leq n$ and $\alpha \equiv 0 \pmod 2$.

$\varphi(y_{\alpha,\beta}) = 1$ if $\beta = 3$.

$\varphi(y_{\alpha,\beta}) = 2$ if $\beta \equiv 0, 2 \pmod 4$ and $\beta \neq 4$.

φ ($y_{\alpha,\beta}$) = 3 if β = 4.
φ ($y_{\alpha,\beta}$) = 4 if β ≡ 1, 3 (mod 4) and β ≠ 3.

Nature of n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$n \equiv 0,1$(modulo 2)	$\frac{n^2+6n}{8}$	$\frac{n^2+6n}{8}$	$\frac{n^2+6n}{8}-1$	$\frac{n^2+6n}{8}$

Table 3.2.9

Nature of n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$n \equiv 0,1$(modulo 2)	$\frac{n^2+6n}{8}-1$	$\frac{n^2+6n}{8}$	$\frac{n^2+6n}{8}-1$	$\frac{n^2+6n}{8}$

Table 3.2.10

The above tables 3.2.9 and 3.2.10 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the graph S ($P_n$ [N_e]) is quotient-4 cordial labeling.

Theorem: 3.2.6 A graph S (T_m) is quotient-4 cordial if m ≥ 3.
Proof: Let G be a S (T_m) graph.
Let V (G) = {u_α : $1 \le \alpha \le$ m} ∪ {x_α : $1 \le \alpha \le$ m-1}∪ { y_α ,v_α : $1 \le \alpha \le$ 2m-4}.
E (G) = {$u_\alpha x_\alpha$: $1 \le \alpha \le m-1$} ∪ { $x_\alpha u_{\alpha+1}$: $1 \le \alpha \le$ m-1} ∪ { $u_\alpha y_\beta$: $2 \le \alpha \le$ m-1, $2\alpha - 3 \le \beta \le 2\alpha - 2$} ∪ { $y_\alpha v_\alpha$: $1 \le \alpha \le 2m-4$}.
Here $|V(G)|$ = 6m – 9, $|E(G)| =$ 6m – 10.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's are given below.
For $1 \le \alpha \le m$.
φ (u_α) = 1 if $\alpha \equiv 0, 1$ (mod 2) and $\alpha \ne 2$.
φ (u_α) = 4 if α = 2.
Labeling of v_α's are given below.
For $1 \le \alpha \le 2m-4$.
φ (v_α) = 2 if $\alpha \equiv 0$ (mod 4) and α = 2.
φ (v_α) = 3 if $\alpha \equiv 3$ (mod 4) and α = 1.
φ (v_α) = 4 if $\alpha \equiv 1, 2$ (mod 4) and $\alpha \ne 1, 2$.
Labeling of x_α's are given below.
For $1 \le \alpha \le m-1$.
φ (x_α) = 3 if $\alpha \equiv 0$ (mod 2).
φ (x_α) = 4 if $\alpha \equiv 1$ (mod 2).
Labeling of y_α's are given below.
For $1 \le \alpha \le 2m-4$.
φ (y_α) = 1 if $\alpha \equiv 1$ (mod 4).
(y_α) = 2 if $\alpha \equiv 0, 2$ (mod 4).
φ (y_α) = 3 if $\alpha \equiv 3$ (mod 4).

Nature of m	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$m \equiv 0$ (modulo 2)	$\frac{3m-4}{2}$	$\frac{3m-4}{2}$	$\frac{3m-4}{2}$	$\frac{3m-4}{2}-1$
$m \equiv 1$ (modulo 2)	$\frac{3m-5}{2}+1$	$\frac{3m-5}{2}$	$\frac{3m-5}{2}$	$\frac{3m-5}{2}$

Table 3.2.11

Nature of m	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$m \equiv 0$ (modulo 2)	$\frac{3m-4}{2}$	$\frac{3m-4}{2}$	$\frac{3m-4}{2}-1$	$\frac{3m-4}{2}-1$
$m \equiv 1$ (modulo 2)	$\frac{3m-5}{2}$	$\frac{3m-5}{2}$	$\frac{3m-5}{2}$	$\frac{3m-5}{2}$

Table 3.2.12

The above tables 3.2.11 and 3.2.12 shows that $|v_\varphi(i) - v_\varphi(j)| \le 1$and $|e_\varphi(k) - e_\varphi(l)| \le 1$. Hence the graph S ($T_m$) is quotient-4 cordial labeling.

Theorem: 3.2.7 A graph S ($P_n \odot K_{1,r}$) is quotient-4 cordial if $r \geq 2$.
Proof: Let G be a S ($P_n \odot K_{1,r}$) graph. Let V (G) = { $u_\alpha : 1 \leq \alpha \leq n$} ∪ { $x_\alpha : 1 \leq \alpha \leq n-1$} ∪ {$y_{\alpha,\beta} : 1 \leq \alpha \leq n, 1 \leq \beta \leq r$ } ∪ {$v_{\alpha,\beta} : 1 \leq \alpha \leq n, 1 \leq \beta \leq r$ } and E (G) = { $u_\alpha x_\alpha$, $x_\alpha u_{\alpha+1} : 1 \leq \alpha \leq n-1$} ∪ { $u_\alpha y_{\alpha,\beta} : 1 \leq \alpha \leq n, 1 \leq \beta \leq r$} ∪ { $y_{\alpha,\beta} v_{\alpha,\beta} : 1 \leq \alpha \leq n, 1 \leq \beta \leq r$}.
Here $|V(G)| = 2n(r+1) - 1$, $|E(G)| = 2n(r+1) - 2$.
Define φ: V (G) → {1, 2, 3, 4}.
Labeling of u_α's are given below.
For $1 \leq \alpha \leq n$.
$\varphi(u_\alpha) = 1$.
Labeling of x_α's are given below.
Case 1:
When $r \equiv 0, 2 \pmod 4$.
For $1 \leq \alpha \leq n$.
$\varphi(x_\alpha) = 4$ if $\alpha \equiv 0 \pmod 2$.
$\varphi(x_\alpha) = 3$ if $\alpha \equiv 1 \pmod 2$.
Case 2:
When $r \equiv 1, 3 \pmod 4$.
For $1 \leq \alpha \leq n$.
$\varphi(x_\alpha) = 3$ if $\alpha \equiv 0, 1 \pmod 2$.
Labeling of $y_{\alpha,\beta}$'s are given below.
Case 1:
When $r \equiv 0 \pmod 4$.
For $\alpha \equiv 1 \pmod 2$.
$\varphi(y_{\alpha,1}) = 4$.
For $1 \leq \alpha \leq n, 2 \leq \beta \leq r$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0, 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 3$ if $\beta \equiv 2, 3 \pmod 4$.
For $\alpha \equiv 0 \pmod 2$
$\varphi(y_{\alpha,1}) = 3$.
For $2 \leq \beta \leq r$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 0, 1 \pmod 4$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 2, 3 \pmod 4$.
Case 2:
When $r \equiv 1 \pmod 4$.
For $1 \leq \beta \leq r - 1$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 1 \pmod 2$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 0 \pmod 2$.
$\varphi(y_{1,r}) = 3$.
$\varphi(y_{\alpha,r}) = 2$ if $2 \leq \alpha \leq n$.
Case 3:
When $r \equiv 2 \pmod 4$.
For $1 \leq \beta \leq r - 2$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 1 \pmod 2$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 0 \pmod 2$.
$\varphi(y_{1,r}) = 4$, $\varphi(y_{1,r-1}) = 1$.
$\varphi(y_{\alpha,r}) = 4$, $\varphi(y_{\alpha,r-1}) = 2$ if $\alpha \equiv 0 \pmod 2$.
$\varphi(y_{\alpha,r}) = 2$, $\varphi(y_{\alpha,r-1}) = 3$ if $\alpha \equiv 1 \pmod 2$ and $\alpha \neq 1$.
Case 4:
When $r \equiv 3 \pmod 4$.
For $1 \leq \beta \leq r - 1$.
$\varphi(y_{\alpha,\beta}) = 1$ if $\beta \equiv 1 \pmod 2$.
$\varphi(y_{\alpha,\beta}) = 4$ if $\beta \equiv 0 \pmod 2$.
$\varphi(y_{1,r}) = 3$.
$\varphi(y_{\alpha,r}) = 2$ if $\alpha \equiv 1, 2 \pmod 2$ and $\alpha \neq 1$.

Labeling of $v_{\alpha,\beta\text{'s}}$ are given below.
Case 1:
When $r \equiv 0$ (mod 4).
For $\alpha = 1$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 2, 3 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 0, 1 \pmod 4$.
For $\alpha \equiv 1$ (mod 2) and $\alpha \neq 1$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 2, 3 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 4$ if $\beta \equiv 0, 1 \pmod 4$ and $\beta \neq 4$.
$\varphi(v_{\alpha,4}) = 2$.
For $\alpha \equiv 0$ (mod 2).
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 2, 3 \pmod 4$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 0, 1 \pmod 4$ and $\beta \neq 4$.
$\varphi(v_{\alpha,4}) = 1$.
Case 2:
When $r \equiv 1$ (mod 4).
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 3$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 0 \pmod 2$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 1 \pmod 2$.
$\varphi(v_{1,r}) = \varphi(v_{1,r-1}) = 2$, $\varphi(v_{1,r-2}) = 3$.
$\varphi(v_{\alpha,r}) = 2$, $\varphi(v_{\alpha,r-1}) = 3$, $\varphi(v_{\alpha,r-2}) = 4$ if $2 \leq \alpha \leq n$.
Case 3:
When $r \equiv 2$ (mod 4).
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 2$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 0 \pmod 2$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 1 \pmod 2$.
$\varphi(v_{1,r}) = 2$, $\varphi(v_{1,r-1}) = 3$.
$\varphi(v_{\alpha,r}) = 4$, $\varphi(v_{\alpha,r-1}) = 2$ if $\alpha \equiv 0 \pmod 2$.
$\varphi(v_{\alpha,r}) = 1$, $\varphi(v_{\alpha,r-1}) = 3$ if $\alpha \equiv 1 \pmod 2$ and $\alpha \neq 1$.
Case 4:
When $r \equiv 3$ (mod 4).
For $1 \leq \alpha \leq n$, $1 \leq \beta \leq r - 3$.
$\varphi(v_{\alpha,\beta}) = 2$ if $\beta \equiv 0 \pmod 2$.
$\varphi(v_{\alpha,\beta}) = 3$ if $\beta \equiv 1 \pmod 2$.
$\varphi(v_{\alpha,r}) = 2$, $\varphi(v_{\alpha,r-1}) = 3$, $\varphi(v_{\alpha,r-1}) = 4$.

Nature of r and n	$v_\varphi(1)$	$v_\varphi(2)$	$v_\varphi(3)$	$v_\varphi(4)$
$r \equiv 0$(modulo 4) $n \equiv 0$(modulo 2)	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$
$r \equiv 0$(modulo 4) $n \equiv 1$(modulo 2)	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4} + 1$
$r \equiv 1$(modulo 4) $n \equiv 0,1$(modulo 2)	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$
$r \equiv 2$(modulo 4) $n \equiv 0$(modulo 2)	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$
$r \equiv 2$(modulo 4) $n \equiv 1$(modulo 2)	$\frac{2n(r+1)-2}{4} + 1$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$
$r \equiv 3$(modulo 4) $n \equiv 0,1$(modulo 2)	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$

Table 3.2.13

Nature of r and n	$e_\varphi(0)$	$e_\varphi(1)$	$e_\varphi(2)$	$e_\varphi(3)$
$r \equiv 0$(modulo 4) $n \equiv 0$(modulo 2)	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$
$r \equiv 0$(modulo 4) $n \equiv 1$(modulo 2)	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$

$r \equiv 1(\text{modulo } 4)$ $n \equiv 0,1(\text{modulo } 2)$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4}$
$r \equiv 2(\text{modulo } 4)$ $n \equiv 0(\text{modulo } 2)$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$
$r \equiv 2(\text{modulo } 4)$ $n \equiv 1(\text{modulo } 2)$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$	$\frac{2n(r+1)-2}{4}$
$r \equiv 3(\text{modulo } 4)$ $n \equiv 0,1(\text{modulo } 2)$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$	$\frac{2n(r+1)}{4}$	$\frac{2n(r+1)}{4} - 1$

Table 3.2.14

The above tables 3.2.13 and 3.1.14 shows that $|v_\varphi(i) - v_\varphi(j)| \leq 1$and $|e_\varphi(k) - e_\varphi(l)| \leq 1$. Hence the graph S ($P_n \odot K_{1,r}$) is quotient-4 cordial labeling.

4.CONCLUSION

In this paper, it is proved that some caterpillar and lobster graphs which admits quotient-4 cordial. The existence ofquotient-4 cordial labeling of different families of graphs will be the future work.

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