**Journal of Advanced Zoology**

*ISSN: 0253-7214*

*Volume 44 Issue S7 Year 2023 Page 1248:1254*

# Seller – Buyer Supply Chain System For Deteriorating Products Involving Floor Space Constraint And Budget Constraint

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***Article History***

*Received:*
*Revised:*
*Accepted*



***Abstract***

An inventory model is developed for a seller -buyer with the supply chain system for deteriorating products. The core objective of the review is reducing the system cost while optimizing the order size and also satisfying the constraints. For this, a Lagrangian multiplier procedure was applied to tackle this sort of non-linear programming mathematical model. This model is illustrated through a numerical example which is easy to computational and took less time too. Further, sensitivity analysis utilized to illustrate the behaviors of developed model.

***Keywords: Inventory, shortages, constraints, Order Quantity, Lagrangian multiplier algorithmic technique***

## 1. INTRODUCTION

It is essential to reduce as much as minimal of deterioration of inventory to optimize the profit and (or) reducing the cost. Notable deterioration may occur during storage period items such as food, drugs, pharmaceuticals, electronic components, chemicals etc., and this loss must be considered when construct the model. This is, therefore managing and a storaging inventory of such products becomes a significance criteria for inventory decision process.

Chiu et al. [2] studied a determinative the best run time for production design with scrap, patch up and unpredictable breakdowns. Jawla and Singh [3] developed multi-thing EPQ model for blemished things with numerous creation arrangements. Karmveer and Ajendra Sharma [4] thought of analysis style concerns for deteriorating things of inventory models. Khannan et al. [5] thought of creation demonstrating for damaged things with blemished assessment strategy, adjust and deals come. Rabbani and Aliabadi [7] cultivated a model with credit worth and selling subordinate interest under tolerable conceded portions and inadequacies. Muniappan et al. [6] created partner EOQ model with stock and items house limit objectives. Ravithammal et al. [8] made accomplice inventory model with stock level need. Vediappan et al. [10] centered on coordination stock model by Lagrange multiplier factor strategy. Uthayakumar and Kumar [9] created inventory model for multi-thing beneath combination of distributions. Cardenas barron et al. [1]

inspected sinking the bourgeois customer amalgamate stock system with mathematics and numerical unsimilarity.

## 2. NOTATIONS AND ASSUMPTIONS

The model uses the following notations and assumptions.

### 2.1 Notations

D Demand rate
$H_v$ Seller's unit holding cost / order
$H_b$ Buyer's unit holding cost / order
$s$ Buyer's unit shortage cost / order
$\mathrm{s_c}$ Seller's unit screening cost / order
$n$ Seller's multiples of order
$\mathrm{R_1}$ Buyer's unit ordering cost / order
$\mathrm{R_2}$ Seller's unit setup cost / order
$\mathrm{Q_c}$ Optimum Order quantity
$\mathrm{Q_1}$ Backorder level
$p$ Buyer's unit purchase cost / order / unit
$F$ Space involved / unit/ item
$X$ Total available storage space for buyer
$W$ Maximum available stock to purchase for buyer

### 2.2 Assumptions

- Consistent demand rate is considered in this model.
- Shortages are going on for buyer as it were.
- Seller screened the damaged product for resale.
- The lot size $Q_c$ satisfies the floor space and budget level requirement. Mathematically, the requirements will be taken as $FQ_c \le X$ and $pQ_c \le W$.

## 3. MODEL FORMULATION

In model formulation, system cost is derived for system development and it determine by the way of the usage of with and without restrictions.

### 3.1 System cost with no constraint

The total cost for buyer and seller is formulated as follows:

$$\mathrm{TC_b} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_1^2}{2Q_c} + \frac{\mathrm{s}(Q_c - Q_1)^2}{2Q_c} \text{ and}$$

$$\mathrm{TC_v} = \frac{\mathrm{DR_2}}{\mathrm{n}Q_c} + \frac{\mathrm{H_v n}Q_c}{2} + \frac{\mathrm{s_c n}Q_c}{2}$$

The system cost is communicated as follows

$$\mathrm{TC_s} = \mathrm{TC_b} + \mathrm{TC_v}$$

$$\mathrm{TC_s} = \frac{\mathrm{DR_1}}{Q_c} + \frac{H_b Q_1^2}{2Q_c} + \frac{\mathrm{s}(Q_c - Q_1)^2}{2Q_c} + \frac{\mathrm{DR_2}}{\mathrm{n}Q_c} + \frac{\mathrm{H_v n}Q_c}{2} + \frac{\mathrm{s_c n}Q_c}{2} \quad (1)$$

Equation (1) can be composed as

$$\mathrm{TC_s} = Q_1^2\left[\frac{\mathrm{s}+H_b}{2\mathrm{Q}}\right] + Q_1[-\mathrm{s}] + \frac{\mathrm{DR_1}}{\mathrm{Q}} + \frac{\mathrm{sQ}}{2} + \frac{\mathrm{DR_2}}{\mathrm{nQ}} + \frac{\mathrm{H_v nQ}}{2} + \frac{\mathrm{s_c nQ}}{2} \quad (2)$$

Equation (2) It is of the structure $a_1 \mathrm{Q}_1^2 + a_2 Q_1 + a_3$.

$Q_1$ will be taken as, $Q_1 = \frac{-a_2}{2a_1}$

$$Q_1 = \frac{sQ}{s+H_b} \tag{3}$$

Equation (3) can be composed as

$$TC_s = Q_c\left[\frac{s+H_v n+s_c n}{2}\right] + \frac{1}{Q_c}\left[DR_1 + \frac{H_b Q_1^2}{2} + \frac{sQ_1^2}{2} + \frac{DR_2}{n}\right] - sQ_1 \tag{4}$$

Equation (4) is of the structure $a_1 Q + \frac{a_2}{Q} + a_3$.

Qwill be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2D\left(R_1+\frac{R_2}{n}\right)(s+H_b)}{n(H_v+s_c)\,(s+H_b)+sH_b}} \tag{5}$$

**3.2 Floor space constraint system cost**

Lagrange multiplier function α is added on system cost because buyer's floor space capacity constraint is considered here.

The system cost with floor space constraint

$$TC_s = TC_b + TC_v + \alpha\,(FQ_c - X)$$

$$TC_s = \frac{DR_1}{Q_c} + \frac{H_b Q_1^2}{2Q_c} + \frac{s(Q_c-Q_1)^2}{2Q_c} + \frac{DR_2}{nQ_c} + \frac{H_v nQ_c}{2} + \frac{s_c nQ_c}{2} + \alpha\,(FQ_c - X) \tag{6}$$

Equation (6) can be composed as

$$TC_s = Q_1^2\left[\frac{s+H_b}{2Q_c}\right] + Q_1[-s] + \frac{DR_1}{Q_c} + \frac{sQ_c}{2} + \frac{DR_2}{nQ_c} + \frac{H_v nQ_c}{2} + \frac{s_c nQ_c}{2} + \alpha FQ_c - \alpha X \tag{7}$$

Equation (7) It is of the structure $a_1 Q_1^2 + a_2 Q_1 + a_3$.

$Q_1$ will be taken as, $Q_1 = \frac{-a_2}{2a_1}$

$$Q_1 = \frac{sQ}{s+H_b} \tag{8}$$

Equation (7) can be composed as

$$TC_s = Q_c\left[\frac{sH_b}{2(s+H_b)} + \frac{n(H_v+s_c)}{2} + \alpha F\right] + \frac{1}{Q_c}\left[D\left(R_1 + \frac{R_2}{n}\right)\right] - \alpha X \tag{9}$$

Equation (9) is of the structure $a_1 Q + \frac{a_2}{Q} + a_3$.

Qwill be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2D\left(R_1+\frac{R_2}{n}\right)(s+H_b)}{sH_b+\{n(H_v+s_c)+2\alpha F\}(s+H_b)}} \tag{10}$$

Where, $\alpha = \frac{2DF^2\left(R_1+\frac{R_2}{n}\right)(s+H_b)-X^2[sH_b+n(H_v+s_c)(s+H_b)]}{2FX^2(s+H_b)}$

**3.3 Budget constraint system cost**

Lagrange multiplier function $\beta$ is added on system cost because buyer's budget constraint is considered here.

The system cost is,

$$TC_s = TC_b + TC_v + \beta\,(pQ_c - W)$$

$$TC_s = \frac{DR_1}{Q_c} + \frac{H_b Q_1^2}{2Q_c} + \frac{s(Q_c - Q_1)^2}{2Q_c} + \frac{DR_2}{nQ_c} + \frac{H_v n Q_c}{2} + \frac{s_c n Q_c}{2} + \beta\,(pQ_c - W) \qquad (11)$$

Equation (11) can be composed as

$$TC_s = Q_1^2\left[\frac{s+H_b}{2Q}\right] + Q_1[-s] + \frac{DR_1}{Q_c} + \frac{sQ_c}{2} + \frac{DR_2}{nQ_c} + \frac{H_v n Q_c}{2} + \frac{s_c n Q_c}{2} + \beta p Q_c - \beta W \quad (12)$$

Equation (13) It is of the structure $a_1 Q_1^2 + a_2 Q_1 + a_3$.

$Q_1$will be taken as, $Q_1 = \frac{-a_2}{2a_1}$

$$Q_1 = \frac{sQ}{s+H_b} \qquad 13)$$

Equation (11) can be composed as

$$TC_s = Q_c\left[\frac{sH_b}{2(s+H_b)} + \frac{n(H_v+s_c)}{2} + \beta p\right] + \frac{1}{Q_c}\left[D\left(R_1 + \frac{R_2}{n}\right)\right] - \beta W \qquad (14)$$

Equation (12) is of the structure $a_1 Q + \frac{a_2}{Q} + a_3$.

Qwill be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2D\left(R_1+\frac{R_2}{n}\right)(s+H_b)}{sH_b+\{n(H_v+s_c)+2\beta p\}(s+H_b)}} \qquad (15)$$

Where, $\beta = \frac{2Dp^2\left(R_1+\frac{R_2}{n}\right)(s+H_b) - W^2[sH_b+n(H_v+s_c)(s+H_b)]}{2pW^2(s+H_b)}$

**3.4 Floor space and budget constraint system cost**

Lagrange multiplier function α and $\beta$ is added on system cost because buyer's floor space constraint and budget constraint is considered here.

The system cost with floor space and budget constraint

$$TC_s = TC_b + TC_v + \alpha\,(FQ_c - X) + \beta\,(pQ_c - W)$$

$$TC_s = \frac{DR_1}{Q_c} + \frac{H_b Q_1^2}{2Q_c} + \frac{s(Q_c - Q_1)^2}{2Q_c} + \frac{DR_2}{nQ_c} + \frac{H_v n Q_c}{2} + \frac{s_c n Q_c}{2} + \alpha\,(FQ_c - X) + \beta\,(pQ_c - W) \qquad (16)$$

Equation (16) can be composed as

$$TC_s = Q_1^2\left[\frac{s+H_b}{2Q}\right] + Q_1[-s] + \frac{DR_1}{Q_c} + \frac{sQ_c}{2} + \frac{DR_2}{nQ_c} + \frac{H_v n Q_c}{2} + \frac{s_c n Q_c}{2} + \alpha F Q_c - \alpha X + \beta p Q_c - \beta W \qquad (17)$$

Equation (17), it is of the structure$a_1 Q_1^2 + a_2 Q_1 + a_3$.

$Q_1$will be taken as, $Q_1 = \frac{-a_2}{2a_1}$

$$Q_1 = \frac{sQ_c}{s+H_b} \qquad (18)$$

Equation (16) can be composed as

$$\mathrm{TC_s} = Q_c\left[\frac{sH_b}{2(s+H_b)} + \frac{n(H_v+s_c)}{2} + \alpha F + \beta p\right] + \frac{1}{Q_c}\left[D\left(R_1 + \frac{R_2}{n}\right)\right] - \alpha X - \beta W \qquad (19)$$

Equation (19) is of the structure $a_1 Q + \frac{a_2}{Q} + a_3$.

Qwill be taken as, $Q = \sqrt{\frac{a_2}{a_1}}$

$$Q_c = \sqrt{\frac{2D\left(R_1+\frac{R_2}{n}\right)(s+H_b)}{sH_b+\{n(H_v+s_c)+2(\alpha F+\beta p)\}(s+H_b)}} \qquad (20)$$

Where, $\alpha = \frac{2DF^2\left(R_1+\frac{R_2}{n}\right)(s+H_b)-X^2[sH_b+\{n(H_v+s_c)+2\beta p\}(s+H_b)]}{2FX^2(s+H_b)}$

Where, $\beta = \frac{2Dp^2\left(R_1+\frac{R_2}{n}\right)(s+H_b)-W^2[sH_b+\{n(H_v+s_c)+2\alpha F\}(s+H_b)]}{2pW^2(s+H_b)}$

**3. 5 Solution Procedure for integrated expected total cost**

**Stage 1.** Track down request amount $Q_c$ and backorder level $Q_1$by (5) and (3). If $Q_c$ satisfies both floor space and budget requirements, then $Q_c$ is the ideal worth to limit the system cost and go to stage 5.

**Stage 2.** Else track down request amount $Q_c$ and backorder level $Q_1$by (10) and (8). If $Q_c$ satisfies floor space requirement, then $Q_c$ is the ideal worth to limit the system cost and go to stage 5.

**Stage 3.** Else track down request amount $Q_c$ and backorder level $Q_1$by (15) and (13). If $Q_c$ satisfies budget requirement, then $Q_c$ is the ideal worth to limit the system cost and go to stage 5.

**Stage 4.** If the above stages are not satisfied then both requirements are active. Now, track down request amount $Q_c$ and backorder level $Q_1$by (20) and (18), then $Q_c$ is the ideal worth to limit the system cost and go to stage 5.

**Stage 5.** End.

## 4. NUMERICAL EXAMPLE

**Example 1**

Let $D = 4000,\ R_1 = 200,\ R_2 = 500,\ H_b = 0.3,\ H_v = 0.5,\ s_c = 0.5,\ p = 0.5,\ s = 1, n = 2,\ F = 3,\ X = 2000,\ W = 300.$

The Optimal solution is,

$Q_c = 500 Q_1 = 384 TC_s = 4.1577\,X\,10^3$satisfies the floor space constraint $FQ_c \leq 2000$ and budget constraint $pQ_c \leq 300$.

### 4.1 Sensitivity Analysis

The sensitivity analysis is completed with the aid of taking each boundary in turn and holding the leftover boundaries unaltered. The impacts are displayed in Table 1.

**Table 1:** Effects of changes in the value of system parameters

| Decision variable | | $Q$ | $Q_1$ | $TC_b$ | $TC_v$ | $TC_s$ |
|---|---|---|---|---|---|---|
| $p$ | $-50$ | 500 | 384 | 1.6577 X10$^3$ | 2.5000X10$^3$ | 4.1577X10$^3$ |
| | $-25$ | 500 | 384 | 1.6577 X10$^3$ | 2.5000X10$^3$ | 4.1577X10$^3$ |
| | $+25$ | 480 | 369 | 1.7221 X10$^3$ | 2.5633X10$^3$ | 4.2854X10$^3$ |
| | $+50$ | 400 | 307 | 2.0462 X10$^3$ | 2.9000X10$^3$ | 4.9462X10$^3$ |
| $n$ | $-50$ | 500 | 384 | 1.6577 X10$^3$ | 4.2500X10$^3$ | 5.9077X10$^3$ |
| | $-25$ | 500 | 384 | 1.6577 X10$^3$ | 3.0417X10$^3$ | 4.6994X10$^3$ |
| | $+25$ | 500 | 384 | 1.6577 X10$^3$ | 2.2250X10$^3$ | 3.8827X10$^3$ |
| | $+50$ | 500 | 384 | 1.6577 X10$^3$ | 2.0833X10$^3$ | 3.7410X10$^3$ |
| $F$ | $-50$ | 600 | 461 | 1.4026X10$^3$ | 2.2667X10$^3$ | 3.6692X10$^3$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| | $-25$ | 600 | 461 | 1.4026X10$^3$ | 2.2667X10$^3$ | 3.6692X10$^3$ |
| | $+25$ | 400 | 307 | 2.0462X10$^3$ | 2.9000X10$^3$ | 4.9462X10$^3$ |
| | $+50$ | 333 | 256 | 2.4385X10$^3$ | 3.3333X10$^3$ | 5.7718X10$^3$ |
| $X$ | $-50$ | 250 | 192 | 3.2288X10$^3$ | 4.2500X10$^3$ | 7.4788X10$^3$ |
| | $-25$ | 375 | 288 | 2.1766X10$^3$ | 3.0417X10$^3$ | 5.2183X10$^3$ |
| | $+25$ | 600 | 461 | 1.4026X10$^3$ | 2.2666X10$^3$ | 3.6692X10$^3$ |
| | $+50$ | 600 | 461 | 1.4026X10$^3$ | 2.2666X10$^3$ | 3.6692X10$^3$ |
| $W$ | $-50$ | 300 | 230 | 2.7013X10$^3$ | 3.6333X10$^3$ | 6.3346X10$^3$ |
| | $-25$ | 350 | 269 | 2.3261X10$^3$ | 3.2071X10$^3$ | 5.5332X10$^3$ |
| | $+25$ | 500 | 384 | 1.6577X10$^3$ | 2.5000X10$^3$ | 4.1577X10$^3$ |
| | $+50$ | 500 | 384 | 1.6577X10$^3$ | 2.5000 X10$^3$ | 4.1577X10$^3$ |
| $s$ | $-50$ | 500 | 312 | 1.6469X10$^3$ | 2.5000 X10$^3$ | 4.1469X10$^3$ |
| | $-25$ | 500 | 357 | 1.6536X10$^3$ | 2.5000 X10$^3$ | 4.1536X10$^3$ |
| | $+25$ | 500 | 403 | 1.6605X10$^3$ | 2.5000 X10$^3$ | 4.1605X10$^3$ |
| | $+50$ | 500 | 416 | 1.6625X10$^3$ | 2.5000 X10$^3$ | 4.1625X10$^3$ |
| $s_c$ | $-50$ | 500 | 384 | 1.6577X10$^3$ | 2.3750X10$^3$ | 4.0327 X10$^3$ |
| | $-25$ | 500 | 384 | 1.6577X10$^3$ | 2.4375 X10$^3$ | 4.0952 X10$^3$ |
| | $+25$ | 500 | 384 | 1.6577X10$^3$ | 2.5625 X10$^3$ | 4.2202 X10$^3$ |
| | $+50$ | 500 | 384 | 1.6577X10$^3$ | 2.6250X10$^3$ | 4.2827 X10$^3$ |

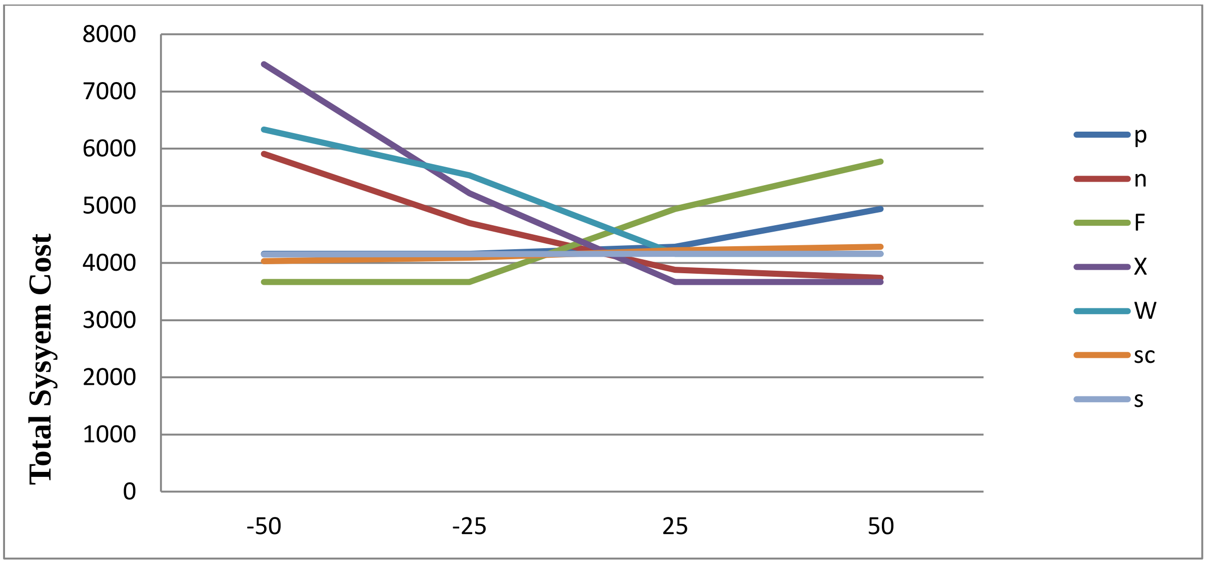


**Fig 1:** Effect of changes when $p, n, F, X, W, s, s_c$ increases

## 5. CONCLUSION

This study revealed a supply chain management consisting of a seller and buyer with the consideration of shortage, screening cost and constraints (floor space and budget constraints). Our model is suitable to solve a problem of optimizing the order size while minimizing system cost. The developed algorithm is simple to understand and it requires nominal times to computation. The similar research findings is also available for multiple-buyer multiple-vendor systems, shortages, postpone installments, exchange credit etc.,

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