Journal of Advanced Zoology

ISSN: 0253-7214
Volume 44 Issue 04 Year 2023 Page 947:949

Alternative Derivations in (1,1) Rings

K. Subhashini[1*], M. Shiva Sankar Reddy[2], V. Raghavendra Prasad[3]

[1.] *Associate Professor, Department of Humanities & Basic Sciences,*
subhashini.gprec@gmail.com
[2,3]*Assistant Professor, Department of Humanities & Basic Sciences mssreddy.bs@gprec.ac.in[2] ;*
raghavendra.bs@gprec.ac.in[3]
G.Pulla Reddy Engineering College, Kurnool-518002, Andhra Pradesh, INDIA

**Corresponding author's E-mail: subhashini.gprec@gmail.com*

Article History

Received: 06 June 2023
Revised: 05 Sept 2023
Accepted: 01 Dec 2023

Abstract

***Objectives**: To show the associativity in one of the subclass of non-associative (γ,δ) rings. **Method:** Derivation alternator rings are limiting case of associative rings. The ring (1,1) is one of the sub class of (γ,δ) rings. Consider a (1,1) derivation alternator ring R, with characteristic ≠ 2, and it is well known that this ring is neither alternative nor flexible. In this paper it will be proved that right alternative property (R,x,x) holds in R and flexibility follows, finally associativity arrives in R. **Findings:** If this ring R does not contain nilpotent elements even though it will be associative. **Novelty:** Further investigators may extend the applications of these rings in science and engineering fields. **Mathematics Subject Classification**: 2010 MSC 17D*

Keywords: *Non associative ring, ideal, center, nucleus, , flexibility, characteristic.*

1. Introduction

Jacobi property $\sum(xy)z = 0$ was proved in derivation alternator Lie rings. by Hentzel, Hogben [1]. Further they established by additional assumption of simplicity, these rings are either alternative or anticommutative. (3) automatically follows (2) in anticommutative rings. K.Suvarna, V.M. Rao [2] showed that a prime derivation alternator ring with $(x, y, z) = (z, y, x)$ is either associative or N = C, the center. Bharathi and Munirathnam [3] proved that idempotent e lies in nucleus in a semi prime derivation alternator ring. "Prime assosymmetric rings are derivation alternator rings". This result was proved by V.M.rao [4] in his Ph.D thesis desertion submitted to S.K.University. This is weaker to K.Subhashini's result [5], in her paper she proved that prime assosymmetric rings are associative. In [6], same author demonstrated that a prime weakly Standard ring is Strongly (-11) ring.

Hemabala [7] emphasized the importance of Γ-near rings in the real-life situations. Non associative rings have been used in lots of communication media from encoding data to coding data to Telecommunications to transmissions to satellites and probes. The importance of rings has been increased in digital world. Rings play vital role in Cryptography, Key exchange, digital signatures etc.

2. Materials And Methods

A derivation alternator ring is a non associative ring and which satisfies the equations

$(y.\ y, y) = 0,$ …..(1)

$(Rz,y,y) = R(z,y,y) + (R, y, y)z,$ …...(2)

And $(y, y, Rz) = R(y, y, z) + (y, y,R)z,$ …...(3)

Where the usual notation for associator is defined as $(y,z,x) = (yz)x - y(zx)$ where as the commutator is $(x, z) = x.z - z.x$. When alternator rings are generalized, then derivation alternator rings were emerged.

(1,1) ring is one of the sub class of (γ,δ) rings having the properties

$\sum(x, y, z) = (x, y, z) + (y, z, x) + (z, x, y) = 0,$ …..(4)

$(y, x, z) - (y, z, x) = 0$ …(5)

In this entire discussion, R represents a (1,1) Derivation Alternator ring with characteristic ≠ 2.

Any arbitrary ring satisfies a common equation given by

$A(u, v, w, x) = (uv, w, x) - (u, vw, x) + (u, v, wx) - u(v, w, x) - (u, v, w)x = 0,$ ...(6)

3. Results and Discussion

Apply (4) to the combination $A(u, v, w, x) - A(v, w, x, u) + A(w, x, u, v) - A(x, u, v, w) = 0$,

Then one gets

$B(u, v, w, x) = (u, (v, w, x)) - (v, (w, x, u)) + (w, (x, u, v)) - (x, (u, v, w)) = 0$ ....(7)

If R is with characteristic $\neq 3$ then

(4) automatically leads to (1). Equation (1) will be discarded.

Replace u and x by v in (7) , add the resulting equation to $(v, \sum(v, v, w))$ and equate to zero, one observes the interesting identity $2(v, (v, w, v)) = 0$. Since characteristic $\neq 2$, so that

$(v, (v, w, v)) = 0$. From (5) and (4) , in (1,1) ring we come across to

$(v, (v, w, v)) = (v,(v, v, w)) = (v,(w, v, v)) = 0$ .....(8)

Let R_R denotes right alternative and written as $R_R = (w, v, v)$. Then from (8) ,

$(v, R_R) = 0$. Commutator is given by $(u, v) = u.v - v.u$. Then

$v.\, R_R = R_R .\, v$

Now replace w by vw in the last equation of (8), implies

$(v, (vw, v, v)) = (v, (v(w, v, v) + (v, v, v)w)$ (from (2))

$0 = (v, v(w, v, v))$ (from (4) with characteristic $\neq 3$)

$= (v, vR_R)$

$= v(R_R.v) - (v.\, R_R)v.$ This implies

$v(R_R.v) = (v.\, R_R)v.$ ....(9)

now take the associator with R_R and from the definition of the associator

$(v, R_R, v) = (v.\, R_R)v - v(R_R.v) = 0,$ (From (9))

From equations (5) & (4) , consequently one can have

$(v, R_R, v) = (v, v, R_R) = (R_R, v, v) = 0.$ Where $R_R = (w, v, v)$ ......(10)

Substitute $w = w^2$ in last part of (10)

$0 = ((w^2, v, v), v, v)$

$= ((w.w, v, v), v, v)$

$= ((w (w, v, v) + (w, v, v) w), v, v)$ *From (2)*

$= ((w.\, R_R + R_R .\, w), v, v)$

$= (w.\, R_R, v, v) + (R_R, v, v)$

$= w(R_R, v, v) + (R_R, v, v)w + R_R .(w, v, v) + (w, v, v).\, R_R,$

$= R_R .\, R_R, + R_R .\, R_R,$

$=2(R_R)^2$

Since R is with characteristic $\neq 2$, gives $(R_R)^2 = 0$. This implies

$(w, v, v)^2 = 0$ implies $(R, v, v)^2=0.$ ....(11)

Theorem: A necessary and sufficient condition that a (1,1) derivation alternator ring R is associative when $(R, v, v)^2=0$.

Proof: If R is associative then automatically equation (11) holds in R.

If $(w, v, v)^2=0$ implies $(w, v, v)=0$ ....(12)

Equation (12) is right alternative.

From (5) and (12)

(v, v, w) = 0, which is left alternative.

And now R is alternative. And hence from (4), R is associative.

4. Conclusion

We draw one conclusion from this theorem that R is associative if nilpotent elements are not present in (1,1) derivation alternator ring.

References:

1. Hentzel.I.R.,Hogben,L., and Smith.H.F. Flexible derivation alternator rings, *Comminications in Algebra*, 1980 ; Vol 8(20) : pp 1997-2014. https://doi.org/10.1080/00927878008822558
2. Suvarna.K, .Rao.V.M Prime antiflexible derivation alternator Rings, *Asian journal of Algebra*,2010, Vol 3 (1), pp 1-7. DOI:10.3923/aja.2010.1.7
3. Bharathi.D, Munirathanam.M Semi prime derivation alternator rings, *International Journal of Scientific & Engineering Research*, 2015, vol 6(9), 116-118. https://www.ijser.org/researchpaper/SEMI-PRIME-DERIVATION-ALTERNATOR-RINGS.pdf
4. Rao.V.M, *Some studies on derivation alternator rings*. Thesis dissertation for Ph.D, 2007: S.K.University Anantapur.
5. Subhashini.K A Note on Assosymmetric Rings, *International Journal of Algebra And Statistics*; 2017, Vol 6(1-2) ,pp 114-116. doi:10.20454/ijas.2017.1222
6. Subhashini.K Theorems on Weakly Standard Rings, *Test Engineering & Management*, 2020, Vol 82 (Jan-Feb), pp 11333-11335. http://testmagzine.biz/index.php/testmagzine/article/view/2675/2352
7. Hemabala.K Multi – Fuzzy Ideals of Γ-near ring, Journal of Advanced Zoology, 2023, Vol 44(S-5), 108-117. DOI:10.17762/jaz.v44iS-5.626