Journal of Advanced Zoology

ISSN: 0253-7214
Volume 44 Issue S-5 Year 2023 Page 1726:1730

Determination of Deflection Function in A Rectangular Body

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Article History

Received: 06 June 2023
Revised: 05 Sept 2023
Accepted: 02 Nov 2023

Abstract

In this work, the authors consider a thick rectangular body, especially a plate, and try to investigate the flow of temperature and deflection function subjected to certain constraints according to their boundaries. Mathematically, the expression of temperature and deflection is found by utilizing standard integral transformation techniques.

Keywords: *Transient problem, temperature distribution, thermal deflection, rectangular body.*

1. Introduction

The study of deflection in various bodies is of great use in various fields that deal with the design and manufacturing of structural materials by considering high heating. Also, the temperature flow presents an accurate and reliable structural analysis of the body, and hence, this type of study may be useful in spacecraft structural design.

The direct thermoelastic modelling by considering rectangular plates studied by Tanigawa and Komatsubara [1], Vihak et al. [2], and Adams and Bert [3] under thermal shock Whereas inverse thermoelastic modelling is framed by Khobragade and Wankhede [4], Khobragade, and Durge [5, 6] under different temperatures and surrounding conditions. Roychaudhari [7] has studied the quasi-static stresses in a thin circular plate due to transient temperature applied along the circumference of a circle over the upper face. Choi et al. [8] discussed the distribution of temperature and the impact of stress under different heating sources by considering rectangular-shaped objects.

This work, deals with study of transient thermoelastic problem to determine the temperature distribution and thermal deflection of the plate occupying the space D: $\{[x,y,z]\in R^3 : -a\le x\le a\,;\, -b\le y\le b\,,\, 0\le z\le h\}$ with the stated boundary conditions. The heat conduction equation is solved with the help of finite Marchi fasulo transform and finite Fourier cosine transform techniques. The results are obtained in the form of infinite series.

Statement of The Problem

Consider a thick isotropic rectangular plate occupying the space D. The differential equation satisfied by the deflection $\omega(\xi,\zeta,t)$ [6]is

$$\nabla^2 M_T(\xi,\zeta,t)+(1-\upsilon)D\nabla^u\omega(\xi,\zeta,t)=0 \tag{1}$$

Where υ is the Poisson's ratio of the material, M_T denote the thermal momentum of the plate and D denote the flexural rigidity,

where $\nabla^2=\dfrac{d^2}{d\xi^2}+\dfrac{d^2}{d\zeta^2}$

And the resultant thermal momentum M_T is defined as

$$M_T(\xi,\zeta,t)=\alpha E\int_o^h z.T(\xi,\zeta,z,t)\,dz \tag{2}$$

Where α and E are the coefficient of liner expansion, Young's modulus respectively. Since the edge of the rectangular plate is fixed and clammed,

$$\frac{\partial^2\omega}{\partial\xi^2}=0,\frac{\partial^2\omega}{\partial\zeta^2}=0 \text{ at } \xi=-a,a\,,\ \zeta=-b,b \text{ and } \frac{\partial\omega}{\partial z}=0 \text{ at } z=0,h \tag{3}$$

The temperature of the plate at time t satisfying the differential equation

$$\frac{\partial^2 T}{\partial\xi^2}+\frac{\partial^2 T}{\partial\zeta^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial T}{\partial t} \tag{4}$$

Where k is the thermal diffusivity to the material of the plate,

Subject to the initial and boundary conditions:

$$T(\xi,\zeta,z,o)=F(\xi,\zeta,z) \tag{5}$$

$$\left[T+k_1\frac{\partial T(\xi,\zeta,z,t)}{\partial\xi}\right]_{\xi=a}=f_1(\zeta,z,t) \tag{6}$$

$$\left[T+k_2\frac{\partial T(\xi,\zeta,z,t)}{\partial\xi}\right]_{\xi=-a}=f_2(\zeta,z,t) \tag{7}$$

$$\left[T+k_3\frac{\partial T(\xi,\zeta,z,t)}{\partial\zeta}\right]_{\zeta=b}=f_3(\xi,z,t) \tag{8}$$

$$\left[T+k_4\frac{\partial T(\xi,\zeta,z,t)}{\partial\zeta}\right]_{\zeta=-b}=f_4(\xi,z,t) \tag{9}$$

$$\left[\frac{\partial T(\xi,\zeta,z,t)}{\partial z}\right]_{z=0}=g(\xi,\zeta,t) \tag{10}$$

$$\left[\frac{\partial T(\xi,\zeta,z,t)}{\partial z}\right]_{z=h}=f(\xi,\zeta,t) \tag{11}$$

Equation (1) to (11) constitute the mathematical formulation of the problem under consideration.

Solution of The Problem

Applying finite Marchi Fasulo transform and finite Fourier cosine transform defined in [10] to the equations (4) to (11) one obtains

$$\frac{\partial\overline{\overline{T}}}{\partial t}+(ka_p^2)\overline{\overline{T}}^*=k\left[(-1)^s\overline{\overline{f}}^*(m,n,t)-\overline{\overline{g}}^*(m,n,t)-\frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s)+\phi\right] \tag{12}$$

Where $a_p^2=a_m^2+b_n^2$

Equation (12) is a first order differential equation whose solution is given by

$$\overline{\overline{T}}^*(m,n,s,t)=k\left[(-1)^s\overline{\overline{f}}^*(m,n,t)-\overline{\overline{g}}^*(m,n,t)-\frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s)+\phi\right]+\overline{\overline{F}}^*(m)e^{-(ka_p^2)t} \tag{13}$$

Where $\overline{T}$ denotes the Marchi Fasulo transform of T and $\overline{\overline{T}}$ denotes the Marchi Fasulo transform of $\overline{T}$, m & n are the Marchi Fasulo transform parameter. $\overline{\overline{T}}^{*}$ denotes the Fourier cosine transform of $\overline{\overline{T}}$ and s is a Fourier cosine transform parameter.

By applying the inversion of Fourier cosine transform and Marchi Fasulo transform to the equation (13), one obtains the expression for temperature distribution as

$$T(\xi,\zeta,z,t)=\sum_{m,n,s=1}^{\infty}\left[\frac{p_m(\xi)}{\lambda m}\right]\left[\frac{p_n(\zeta)}{\lambda n}\right]\left\{\begin{array}{l}\left[k(c-1)^s\overline{\overline{f}}(m,n,t)-\overline{\overline{g}}(m,n,t)-\frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s)+\varphi\right]\\+\left[\frac{2}{h}\overline{\overline{F}}_c(s)\times\cos\left(\frac{s\pi z}{h}\right).e^{-(ka_p^2)t}\right]\end{array}\right\} \tag{14}$$

Were

$$\overline{\overline{f}}(m,n,t)=\int_{-a}^{a}\overline{f}(m,z,t)\,P_m(\xi)\,d\xi\,,$$

$$\overline{\overline{g}}(m,n,t)=\int_{-a}^{a}\overline{g}(m,z,t)\,P_m(\xi)\,d\xi\,,$$

$$\lambda_m=\int_{-a}^{a}P_m^2(\xi)\,d\xi$$

$$P_m(\xi)=Q_m\cos(a_m\xi)-W_m\sin(a_m\xi)$$

$$Q_m=a_m(\alpha_3+\alpha_4)\cos(a_m a)+(\beta_3-\beta_4)\sin(a_m a)$$

$$W_m=(\beta_3+\beta_4)\cos(a_m a)+(\alpha_4-\alpha_3)\,a_m\sin(a_m a)$$

Equation (14) is the desired solution of the given problem with $\beta_3=\beta_4=1$, $\alpha_3=k_3,\ \alpha_4=k_4$.

Determination Of Thermal Deflection

Substituting the value of temperature distribution $T(\xi,\zeta,z,t)$ from equation (14) to the equation (2) we obtain the expression for thermal momentum as

$$M_T=K\alpha E\sum_{m,n,s=1}^{\infty}\left[\frac{p_m(\xi)}{\lambda m}\right]\left[\frac{p_n(\zeta)}{\lambda n}\right]$$

$$\left\{\left\{(-1)^s\left[\frac{2}{h}\overline{\overline{F}}_c(s)e^{-(ka_p^2)t}\left(\frac{s\pi}{h}\right)^2\right]+\overline{\overline{f}}(m,n,t)\right\}-\overline{\overline{g}}(m,n,t)-\frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s)+\varphi\right\} \tag{15}$$

We assume the solution of equation (1) satisfying condition (3) as

$$\omega(\xi,\zeta,t)=\sum_{m=1}^{\infty}c_m(t)\xi\zeta\sin\left(\frac{m\pi z}{h}\right).[z(z-h)] \tag{16}$$

It can be easily seen that

$$\frac{\partial^2\omega}{\partial\xi^2}=0,\frac{\partial^2\omega}{\partial\zeta^2}=0 \text{ at } \xi=-a,\,a\,,\ \zeta=-b,\ b \text{ and } \frac{\partial\omega}{\partial z}=0 \text{ at } z=0,h$$

Hence solution (16) satisfies the condition (3). Now,

$$\nabla^4\omega(\xi,\zeta,t)=8\sum_{m=1}^{\infty}c_m(t)\sin\left(\frac{m\pi z}{h}\right).[z(z-h)] \tag{17}$$

and

$$\nabla^2 M_T = K\alpha E \sum_{m,n,s=1}^{\infty}\left[\frac{p''_m(\xi)}{\lambda_m}\right]\left[\frac{p''_n(\zeta)}{\lambda_n}\right]$$

$$\times\frac{1}{D(1-\upsilon)}\left\{\left[(-1)^s \overline{\overline{f}} - \overline{\overline{g}} - \frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s) + \varphi\right] + \left[\frac{2}{h}\overline{\overline{F}}_c\ e^{-(ka_p^2).t}\left(\frac{s\pi}{h}\right)^2(-1)^s\right]\right\} \tag{18}$$

Comparing equations (15) and (16), one obtains

$$C_m(t) = \frac{K\alpha E}{8}\sum_{m,n,s=1}^{\infty}\left[\frac{p''_m(\xi)}{\lambda_m}\right]\left[\frac{p''_n(\zeta)}{\lambda_n}\right]\frac{1}{\sin\left(\frac{m\pi}{n}\right).z[z(z-h)]}$$

$$\times\left[\frac{1}{D(1-\upsilon)}\right]\left[(-1)^s \overline{\overline{f}} - \overline{\overline{g}} - \frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s) + \frac{2}{h}\overline{\overline{F}}_c\ e^{-(ka_p^2).t}\times\left(\frac{s\pi}{h}\right)^2(-1)^s\right] \tag{19}$$

Substituting equation (18) in equation (15), we get

$$\omega(x,y,t) = \frac{K\alpha E}{8D(1-\upsilon)}\sum_{m,n,s=1}^{\infty}\left[\frac{p''_m(\xi)}{\lambda_m}\right]\left[\frac{p''_n(\zeta)}{\lambda_n}\right]a^2b^2$$

$$\times\left[(-1)^s \overline{\overline{f}} + \frac{2}{h}\overline{\overline{F}}_c\ e^{-(ka_p^2).t}\left(\frac{s\pi}{h}\right)^2 - \overline{\overline{g}} - \left(\frac{s\pi}{h}\right)^2 Tc^{(s)}\right] \tag{20}$$

where

$P''_m(\xi) = \nabla^2 P_m(\xi)$, $P''_n(\zeta) = \nabla^2 P_n(\zeta)$.

Special Case and Numerical Results

Setting

$$f(\xi,\zeta,t) = (1-\overset{-t}{e})(\xi+a)^2(\xi-a)^2\ (\zeta-b)^2(\zeta+b)^2\ e^h\ (1+c)$$

and $$g(\xi,\zeta,t) = (1-\overset{-t}{e})(\xi+a)^2(\xi-a)^2\ (\zeta+b)^2(\zeta-b)^2\ (1+c),$$

$$\beta = 28.132\,k(k_1+k_2)(k_2+k_3)(1+C).,$$

$$\delta = \frac{28.132\,k(k_1+k_2)(k_2+k_3)\alpha E\,(1+C).}{8SD(1-\upsilon)},\quad a=4, b=2, h=1, t=1\sec, k=0.86$$

in equation (14) and (20), one obtain the expressions for temperature distribution and thermal deflection as

$$\frac{T(\xi,\zeta,z,t)}{\beta} = \sum_{m,m,s=1}^{\infty}\left[\frac{P_m(\xi)}{\lambda_m}\right]\left[\frac{P_n(\zeta)}{\lambda_n}\right]\left[(-1)^s - 1\right]\times\left[\frac{4a_n\cos^2(4a_n) - \cos(4a_n)\sin(4a_n)}{a_n{}^2}\right]$$

$$\times\left[\frac{2b_m\cos^2(2b_m) - \cos(2b_m).\sin(2b_m)}{b_m{}^2}\right]$$

$$+\frac{2}{1+(9.8596)s^2}\left[e^1(-1)^s - 1\right].\cos(3.145).ze^{-k(a_n^2+b_m^2)} \tag{21}$$

$$\frac{\omega(\xi,\zeta,t)}{\delta} = \sum_{m,n,s=1}^{\infty}\left[\frac{p''_m(\xi)}{\lambda_m}\right]\left[\frac{p''_n(\zeta)}{\lambda_n}\right](a^2.b^2)\left[\frac{4a_n\cos^2(4a_n)-\cos(4a_n)\sin(4a_n)}{a_n{}^2}\right]$$

$$\times\left[\frac{2b_m\cos^2(2b_m)-\cos(2b_m).\sin(2b_m)}{b_m{}^2}\right].\left[(-1)^s-1\right]$$

$$+19.7192\left[\frac{e^1(-1)^s-1}{1+(9.8596).s^2}\right]s^2.e^{-k(a_n^2+b_m^2).} \qquad (22)$$

4. Conclusion

The temperature distribution and thermal deflection at any point of a thick rectangular plate have been derived when the boundary conditions are known; with the aid of Marchi-Fasulo transform and finite Fourier cosine transform techniques. The series solution is convergent. The temperature distribution and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications

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