



Bianchi Type V Cosmological Scenario In $f(R, T)$ Gravity Theory With Special Form Of Scale Factor

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Abstract:

In this study, we have examined the exact solutions of the field equations for a Bianchi type V universe filled with bulk viscous fluid within the framework of $f(R, T)$ theory, where $f(R, T) = R + 2f(T)$ and R and T represent the Ricci scalar and the trace of the energy momentum tensor, respectively. We used a combination of exponential and hyperbolic scale factors to determine the physical parameters and metric potentials in the space-time. We also investigated the geometrical and physical parameters of the model, as well as the energy conditions. Additionally, we found that the state finder diagnostic pair falls within an acceptable range.

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Keywords: $f(R, T)$ gravity, Bianchi type V, Bulk viscous fluid, deceleration parameter.

1. INTRODUCTION

According to the cosmological observations, modern cosmology attracts much attention of the researchers because of its ability to explain the late-time acceleration of the Universe. One possibility in explaining the observations is by assuming that at large scales the Einstein gravity model of general relativity breaks down, and a more general action describes the gravitational field. This is the main reason why the modern cosmology is the fastest growing field in the study of the Universe. Modern cosmology achieved a new path because of the idea of accelerated expansion of the Universe. This idea was observed by type-Ia supernovae experiments, suggesting that the Universe is undergoing an accelerated expansion [1–6]. As a result of the coupling the motion of the massive particles becomes non geodesic and an extra-force orthogonal to the four velocities arises. The connections with modified Newtonian dynamics and the pioneer anomaly were explored. This model was extended to the case of arbitrary coupling in both geometry and matter in [7]. The astrophysical and cosmological implications of the non minimal coupling matter-geometry coupling were extensively investigated in [8, 9]. The Palatini formulation of the non minimal geometry-coupling models was considered in [10]. In this context a maximal extension of the Hilbert-Einstein action was proposed [11] by assuming that the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the matter Lagrangian L_m . The gravitational field equations have been obtained in the metric formalism as well as the equations of motion for test particles, following from the covariant divergence of the stress energy tensor. Harko et al [12] proposed $f(R, T)$ gravity theory by taking into account the gravitational Lagrangian as the function of Ricci

scalar R and of the trace of energy-stress tensor T . They have obtained the equation of motion of test particle and the gravitational field equation in metric formalism both.

Recently several cosmological models have been developed in $f(R, T)$ gravity in framework of non-exotic matter that give the clue that the trace of energy momentum-tensor may be responsible for present cosmic acceleration in the universe [13–21]. The bulk viscosity in $f(R, T)$ gravity for a FRW universe is introduced in [22]. They have studied the realistic models considering the dissipative processes due to the presence of viscosity. Later, the bulk viscous cosmological model for anisotropic Bianchi I universe in this theory was presented in [23]. The authors in [24] have studied the dynamics of shearing anisotropic viscous fluid and its stability with cylindrical symmetry in $f(R, T)$ gravity. Moreover, the Little Rip and Big Rip model in $f(R, T)$ theory of gravity was investigated in [25–27]. The bouncing scenario of the $f(R, T)$ gravity model was well explained by Singh et al [28]. Aktas and Aygün [29] have discussed magnetised strange quark matter solutions in $f(R, T)$ gravity with a cosmological constant and they found that $f(R, T)$ theory can explain the late-time acceleration of the Universe. Samanta and Myrzakulov [30] studied bulk viscous fluid in $f(R, T)$ theory. Very recently, Pawar et al [31] have discussed the Bianchi-V model in the presence of $f(R, T)$ gravity using modified holographic Ricci dark energy and they found negative value of the deceleration parameter (DP) which indicates that the Universe is in the accelerated expansion phase and they observed that the Universe is isotropic throughout the evolution. Similarly, Pawar et al [32] and Sharif [33] have analysed $f(R, T)$ theory with different energy sources and in different cosmological models. Samanta [34] has investigated $f(R, T)$ gravity for the Bianchi type-V Universe filled with wet dark fluid.

Motivated by the above discussion, in the present paper, we consider spatially homogeneous and anisotropic Bianchi type-V universe filled with bulk viscous fluid cosmological model in the $f(R, T)$ theory of gravity. The geometrical and physical aspects of the models are also studied. This work aims to investigate new class of Bianchi type V bulk viscous fluid cosmological model under $f(R, T)$ gravity and it is organized as follow. The paper is organised as follows: Section 2 discusses gravitational field equation of $f(R, T)$ modified gravity. In section 3, we have studied the metric (Bianchi type-V) and field equations for $f(R, T)$ gravity. In section 4, we have discussed Dynamical parameters and their physical discussion. Section 5 is devoted to the cosmological interpretations. Finally, in section 6, we have concluded our work.

2. GRAVITATIONAL FIELD EQUATION OF $f(R, T)$ MODIFIED GRAVITY

The $f(R, T)$ theory of gravity is the generalization or modification of General Relativity (GR). In this theory, the modified gravity action is given by

$$S = \int \left[\frac{1}{2\kappa} f(R, T) + L_m \right] \sqrt{-g} d^4x, \quad (1)$$

where $f(R, T)$ is an arbitrary function of the Ricci scalar R , T is the stress energy tensor T_{ij} of matter and L_m is the matter Lagrangian density. It would be worthwhile to mention that if we replace $f(R, T)$ with $f(R)$, we get the action for $f(R)$ gravity and the displacement of $f(R, T)$ with R leads to the action of GR. g is the determinant of the metric tensor g_{ij} . The $f(R, T)$ gravity field equations are obtained by varying the action S in equation (1) with respect to the metric tensor

$$f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} - (\nabla_i \nabla_j - g_{ij}\Pi)f_R(R, T) = \kappa T_{ij} - f_T(R, T)\left(T_{ij} - \frac{1}{3}\theta_{ij}\right). \quad (2)$$

where ∇_i being the covariant derivative and

$$\Pi = \nabla^i \nabla_i f_R = \frac{\partial f(R, T)}{\partial R} \quad \text{and} \quad f_T = \frac{\partial f(R, T)}{\partial T} \quad \theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}} \quad (3)$$

The field equations in $f(R, T)$ modified gravity model we assume that the particular functional $f(R, T)$ as

$$f(R, T) = R + 2f(T) \quad (4)$$

Otherwise functional can be taken in different ways corresponding to viable models. Here $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter.

By using this functional, field equation can be rewritten as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2\bar{p}f'(T) + f(T)]g_{ij}. \quad (5)$$

where the prime denotes a derivative with respect to the argument.

The simplest cosmological model can be obtained by choosing the function $f(T)$ so that $f(T) = \lambda T$, where λ is a constant.

3. METRIC AND FIELD EQUATIONS

Now we consider a The diagonal form of the metric of Bianchi type V universe with the metric as

$$ds^2 = dt^2 - A^2dx^2 - e^{2\beta x}[B^2dy^2 + C^2dz^2] \quad (6)$$

Here A, B, C are cosmic scale factors and β is an arbitrary constant.

Moreover, assuming the energy-momentum tensor for an imperfect bulk viscous fluid which takes the form

$$T_{ij} = (\rho + \bar{p})u_iu_j - \bar{p}g_{ij} \quad (7)$$

where \bar{p} is the effective pressure given by

$$\bar{p} = p - \zeta u^i{}_{;i} \quad (8)$$

satisfying a linear equation of state

$$p = \epsilon\rho, \quad 0 \leq \epsilon \leq 1. \quad (9)$$

Here p is the equilibrium pressure, ρ is the energy density of matter, ζ is the coefficient of bulk viscosity and u^i is the flow vector of the fluid satisfying $u_iu^i = 1$. The semicolon stands for covariant differentiation. On thermodynamic grounds bulk viscosity coefficient ζ is positive, assuring that the viscosity pushes the dissipative pressure \bar{p} towards negative values. However, the correction applied to the thermodynamical pressure p due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution are not fundamentally influenced by the inclusion of the viscous term in the energy-momentum tensor.

The spatial volume V and the average Hubble's parameter H are defined as

$$V = a^3 = ABC, \quad (10)$$

$$3H = \frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (11)$$

The shear scalar σ and anisotropy parameter Am are defined as follows

$$\sigma^2 = \frac{1}{2} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \theta^2 \quad (12)$$

$$Am = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 \quad (13)$$

where $\Delta H_i = H_i - H$, ($i = 1, 2, 3$) and $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\beta^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p} \quad (14)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (15)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (16)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\beta^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (17)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (18)$$

After integrating eq. (18) and absorbing integration constant into B or C , we get

$$A^2 = BC \quad (19)$$

These we have five highly non-linear differential equations with six unknowns, namely $A, B, C, \rho, \bar{p}, \zeta$. Therefore to find a consistent solution to these equations, subtracting (16) from (15), eq. (17) from (16), eq. (17) from (15) and integrating the resulting equations, we obtain the following three relations respectively:

$$\frac{A}{B} = m_1 \operatorname{dexp} \left[k_1 \int \frac{dt}{a^3} \right] \quad (20)$$

$$\frac{A}{C} = m_2 \operatorname{dexp} \left[k_2 \int \frac{dt}{a^3} \right] \quad (21)$$

$$\frac{B}{C} = m_3 \operatorname{dexp} \left[k_3 \int \frac{dt}{a^3} \right] \quad (22)$$

where $m_1, m_2, m_3, k_1, k_2, k_3$ are constants of integration.

We write the metric functions from (20)-(22) in explicit form as

$$A = ad_1 \operatorname{dexp} \left[\alpha_1 \int \frac{dt}{a^3} \right] \quad (23)$$

$$B = ad_2 \operatorname{dexp} \left[\alpha_2 \int \frac{dt}{a^3} \right] \quad (24)$$

$$C = ad_3 \operatorname{dexp} \left[\alpha_3 \int \frac{dt}{a^3} \right] \quad (25)$$

$$\text{where } d_1 = \sqrt[3]{m_1 m_2}, \quad d_2 = \sqrt[3]{m_1^{-1} m_3}, \quad d_3 = \sqrt[3]{(m_2 m_3)^{-1}}, \quad (26)$$

$$\text{and } \alpha_1 = \frac{k_1 + k_2}{3}, \quad \alpha_2 = \frac{k_3 - k_1}{3}, \quad \alpha_3 = \frac{-(k_2 + k_3)}{3}, \quad (27)$$

The constants d_1, d_2, d_3 and $\alpha_1, \alpha_2, \alpha_3$ satisfy the following two relations:

$$\alpha_1 + \alpha_2 + \alpha_3 = 0, \quad d_1 d_2 d_3 = 1. \quad (28)$$

Substituting eq. (19) in eqs. (23)-(25), we obtain

$$A = a \quad (29)$$

$$B = ad \operatorname{dexp} \left[\alpha \int \frac{dt}{a^3} \right], \quad (30)$$

$$C = ad^{-1} \operatorname{dexp} \left[-\alpha \int \frac{dt}{a^3} \right], \quad (31)$$

$$\text{where } d_1 = 1, d_2 = d_3^{-1} = d, \quad \alpha_1 = 0, \quad \alpha_2 = -\alpha_3 = \alpha$$

4. DYNAMICAL PARAMETERS AND THEIR PHYSICAL DISCUSSION

The cosmological parameters such as scale factor a , Hubble parameter H , deceleration parameter q * have a very significant role in describing the evolution of the Universe. And, these are the key parameters of most of the cosmological models in modified gravity theories. The modified gravity field equations can be solved by considering explicit form of the average scale factor as

$$a = e^{nt} \operatorname{sech} mt \quad (32)$$

The derivation and the motivation to choose such scale factor has already been described by Moraes and Santos [35]

The spatial volume of the metric is

$$V = a^3 = e^{3nt} (\operatorname{sech} mt)^3 \quad (33)$$

Substituting (32) in (29)-(31) and integrating, we obtain expression for the metric functions as

$$A = e^{nt} \operatorname{sech} mt \tag{34}$$

$$B = d \operatorname{sech}(mt) \exp[nt + kpe^{-3nt} \cosh(3mt) - qcosh(mt) + 2m \sinh(mt)(rcosh(2mt) + s)] \tag{35}$$

$$C = d^{-1} \operatorname{sech}(mt) \exp[nt - kpe^{-3nt} \cosh(3mt) - qcosh(mt) + 2m \sinh(mt)(rcosh(2mt) + s)] \tag{36}$$

where $\frac{\alpha}{12(9n^4 - 10n^2m^2 + m^4)} = k$, $nm^2 - 9n^3 = p$, $27n(n^2 - m^2) = q$,

$$m^2 - 9n^2 = r, \quad 5m^2 - 9n^2 = s$$

The directional Hubble parameters are

$$H_1 = \frac{\dot{A}}{A} = n - m \tanh(mt) \tag{37}$$

$$H_2 = \frac{\dot{B}}{B} = n + 3kpe^{-3nt} [m \sinh(3mt) - n \cosh(3mt)] - mq \sinh(mt) + 4m^2 r \sinh(mt) \sinh(2mt) + 2m^2 \cosh(mt)(rcosh(2mt) + s) \tag{38}$$

$$H_3 = \frac{\dot{C}}{C} = n - 3kpe^{-3nt} [m \sinh(3mt) - n \cosh(3mt)] - mq \sinh(mt) + 4m^2 r \sinh(mt) \sinh(2mt) + 2m^2 \cosh(mt)(rcosh(2mt) + s) \tag{39}$$

The average Hubble parameter is

$$H = \frac{1}{3} [3n - m \tanh(mt)] \tag{40}$$

The dynamical scalar expansion θ and shear scalar σ^2

$$\theta = [3n - m \tanh(mt)] \tag{41}$$

$$\sigma^2 = \left\{ \frac{1}{3} m^2 \tanh^2(mt) + [3kpe^{-3nt} [m \sinh(3mt) - n \cosh(3mt)] - mq \sinh(mt) + 4m^2 r \sinh(mt) \sinh(2mt) + 2m^2 \cosh(mt)(rcosh(2mt) + s)]^2 \right\} \tag{42}$$

The average anisotropic parameter Am is

$$Am = \frac{1}{3} \left\{ \frac{[n - m \tanh(mt)]^2 + 2n^2 + 2[3kpe^{-3nt} [m \sinh(3mt) - n \cosh(3mt)] - mq \sinh(mt) + 4m^2 r \sinh(mt) \sinh(2mt) + 2m^2 \cosh(mt)(rcosh(2mt) + s)]^2}{\frac{1}{9} [3n - m \tanh(mt)]^2} + 1 \right\} \tag{43}$$

The deceleration parameter is

$$q^* = -1 - \frac{m^2}{\cosh(mt)[3n \cosh(mt) - m \sinh(mt)]} \tag{44}$$

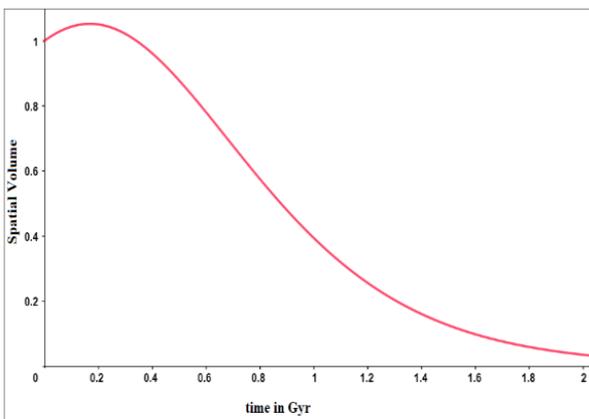


Figure 1. Spatial Volume vs. time for $m=1.1$, $n=0.2$

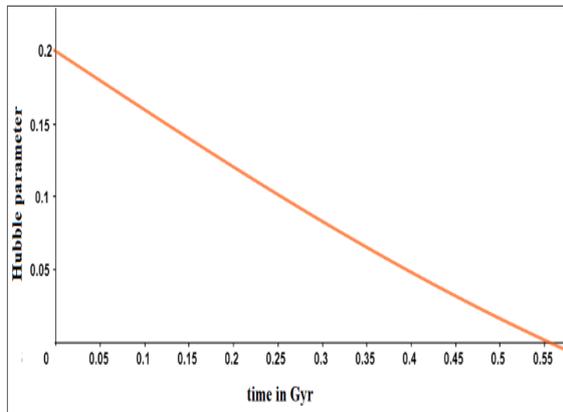


Figure 2. Hubble Parameter vs. time For $m=1.1$, $n=0.2$

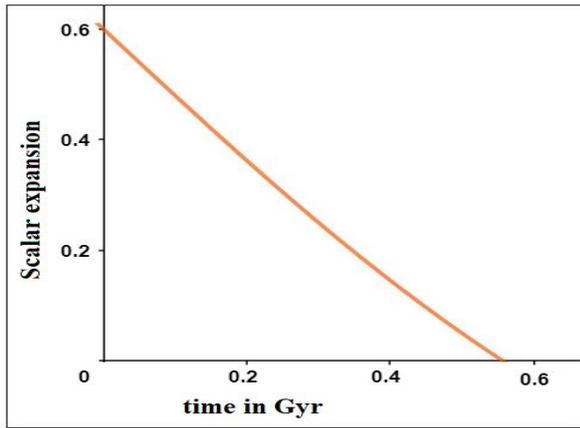


Figure 3. Scalar expansion vs. time $m=1.1$, $n=0.2$

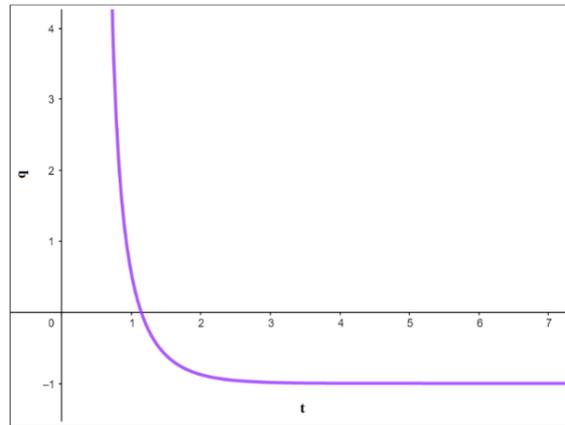


Figure 4. Plot of deceleration parameter q as a function of cosmic time t for $m = 1.1, n = 0.2$

For the model represented by metric functions in (34)-(36) the energy density ρ and the bulk viscous pressure \bar{p} are given by

$$\rho = \frac{1}{(8\pi+2\lambda)} \left\{ n^2 - [3kpe^{-3nt} [m \sinh(3mt) - n \cosh(3mt)] - mqsinh(mt) 4m^2rsinh(mt)sinh(2mt) + 2m^2cosh(mt)(rcosh(2mt) + s)]^2 + 2n \frac{(8\pi+3\lambda)}{(8\pi+4\lambda)} [n - mtanh(mt)] - \frac{4n^2\lambda}{(8\pi+4\lambda)} - 8 \frac{(3\pi+\lambda)\beta^2}{(8\pi+4\lambda)} e^{-2nt} cosh^2 mt \right\} \quad (45)$$

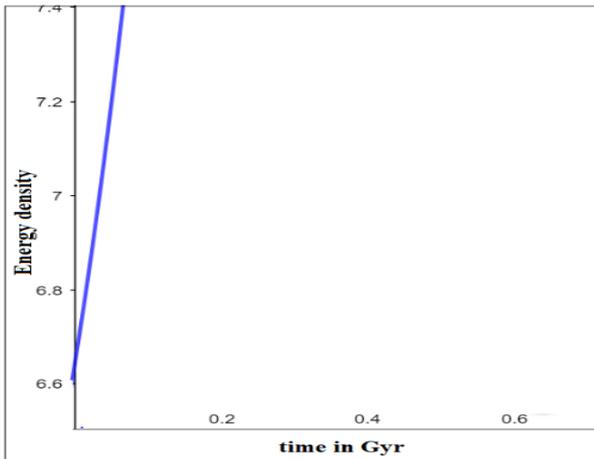


Figure 5. Plot of Energy density ρ vs. time t for $m = 1.1, n = 0.2, \alpha = \beta = \lambda = 1$

$$\bar{p} = \frac{1}{(8\pi + 2\lambda)(8\pi + 4\lambda)} \{ 2n\lambda(n - mtanh(mt)) - (8\pi + 2\lambda) [n - mqsinh(mt) + 4m^2rsinh(mt)sinh(2mt) + 2m^2cosh(mt)(rcosh(2mt) + s)]^2 - 9(kp)^2 e^{-6nt} [m^2 \sinh^2(3mt) + n^2 \cosh^2(3mt) - mnsinh(6mt)] + 8\pi\beta^2 e^{-2nt} cosh^2 mt \} \quad (46)$$

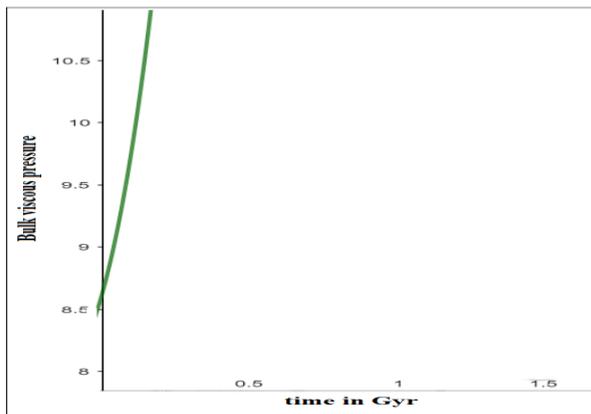


Figure 6. Plot of Bulk viscous pressure \bar{p} vs. time t for $m = 1.1, n = 0.2, \alpha = \beta = \lambda = 1$

The barotropic equation of state parameter may be used to obtain the coefficient of bulk viscosity, which is obtained from Eqs. (46) as

$$\zeta = \frac{1}{(8\pi + 2\lambda)(8\pi + 4\lambda)(3n - mtanh(mt))} \{ [8\pi(\epsilon + 1) + 2(2\epsilon + 1)\lambda] [n - mqsinh(mt) + 4m^2rsinh(mt)sinh(2mt) + 2m^2cosh(mt)(rcosh(2mt) + s)]^2 - 9(kp)^2 e^{-6nt} [m^2 sinh^2(3mt) + n^2 cosh^2(3mt) - mnsinh(6mt)] + [16n\pi + 2n\lambda(3\epsilon - 1)](n - mtanh(mt)) - 4n^2\lambda\epsilon - 8[\epsilon(3\pi + \lambda) + \pi]\beta^2 e^{-2nt} cosh^2 mt \} \quad (47)$$

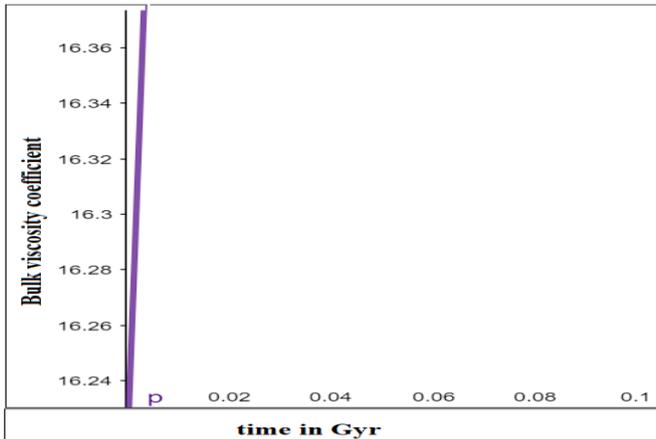


Figure 7. Plot of Bulk viscosity Coefficient ζ vs. time t for $m = 1.1, n = 0.2, \alpha = \beta = \lambda = 1, \epsilon = 0.1$

5. COSMOLOGICAL INTERPRETATIONS

We have the following observations. Recall that our aim in this work is to enable $f(R, T)$ gravity to induce a cosmological scenario.

- 1) All the scale factors and spatial volume (V) is constant at $t = 0$. This shows that the universe starts evolving with constant volume at $t = 0$ and expands with cosmic time t . The spatial volume (V) is finite at initial epoch and it decreases with increase in cosmic time (Fig. 1).
- 2) The mean Hubble’s parameter H and the directional Hubble’s parameters are dynamical. From Fig. 2, one can easily see that the Hubble parameter H is a decreasing function over the growth of time.
- 3) Initially scalar expansion (Fig. 3) is finite. It decreases as cosmic time increases.
- 4) The evolution of the deceleration parameter as a function of cosmic time presented in Fig. 4. From Fig. 4, one can observe that the evolution of the deceleration parameter starts with positive value of q^* , which represents the deceleration phase. it gradually decreases and maintains a constant behaviour during the late time of the universe. It shows universe is accelerating. After late time, it goes to the de-Sitter expansion phase.
- 5) Energy density ρ , Bulk Viscous Pressure \bar{p} and Bulk viscosity Coefficient ζ (Fig. 5, 6, 7) increases with time. Energy density and Bulk viscous pressure are constant at an initial time and it tends to infinity as t tends to infinity.
- 6) We observed that, initially the positive value of the Hubble parameter and the deceleration parameter shows that the universe is expanding and accelerating exponentially at early time. Our model is expanding, shearing and accelerating and has no initial singularity.

6. CONCLUSION

In this article, we have constructed a cosmological scenario of Bianchi type V universe in $f(R, T)$ gravity. The gravitational field equation has been established by taking $f(R, T) = R + 2f(T)$ into consideration. To find the deterministic solution, we have considered a scale factor as $a = e^{nt} sechmt$, where n and m are positive constants. This generates a transition of the universe from the early decelerating phase to the recent accelerating phase.

The main features of the models are as follows:

- The models are based on exact solutions of the $f(R, T)$ gravity field equations for the anisotropic Bianchi-V space-time filled with bulk viscous fluid.
- The energy density has been graphed versus time in Fig. 4. It is evident that the energy density remains always positive and increasing function of time. Initially it is constant for $t = 0$.
- From the figure we observe that bulk viscous pressure is increasing function of time. It starts from a constant positive value and approaches to infinity.
- At the initial time, we have finite energy density, finite bulk viscous pressure and as we discussed earlier. This means that our Universe has no initial singularity.
- We observe that the model has no initial singularity at $t = 0$. Also, we see that H, θ are finite at $t = 0$. These parameters are decreasing function of time. Whereas initially for $t = 0$, the parameters ρ, \bar{p}, ζ are constant and increasing function of time and approaches to infinity.
- The model represents an expanding, shearing, non-rotating and accelerating universe.
- Now for a Universe which was decelerated in past and accelerating at present epoch, the DP must show signature flipping as already discussed. Therefore, our consideration of DP to be variable is physically justified.

Our derived model is accelerating at present epoch

Thus, the solutions demonstrated in this paper may be useful for better understanding of the scenario of Bianchi type V universe in the evolution of the universe within the framework of $f(R, T)$ gravity theory.

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