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(M/EK/C:C/FCFS) Queue with Two-Class Arrivals,State dependent service, Multi-Servers, and Customers Impatience

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Abstract

This paper deals with the finite Markovian queueing system with multi server facility, customer impatience and two types of arrivalswith heterogeneous arrival ratesalong withphase wise state dependent service. Arrival and departures assumed to be Poisson and exponential distributionsrespectively under first-come-first-serve basis. The service will be rendered in k-phases. Transient state probabilities as well as some performance indices have been calculated.We have presented the results of sensitivity analysis and observed the impact ofvarious parameters on system's constants.

Key words: Two-class customers, Balking, Renegingand multi-server facility

I Introduction

Queueing theory is a part of Operations Research that is used for drawing optimum solutions to provide service with the given constraints. Queues help organisations to render service in an orderly manner. The analysis is about fixing a mathematical model with the given arrival and service rates. Recent times are witnessing great attention in queueing models due to their wide applications.

Haight [6] was the first to investigate the concept of customer impatience. He considered a balking model for M/M/1 queues in which an arrival would not balk at the longest queue length. Balking (refusing to join the queue immediately upon the arrival) and Reneging were two of the Queuing Problems detailed by Ancker and Gafarian [2]. Later many researchers[1,3,13,15,16] have made significant contribution in this aspect.

Many researchers have focussed on techniques that can reduce customer impatience. Concept of multi-servers is one among them. Y.Levi et.al[18] have studied and obtained performance

measures for an M/M/s queue with vacation. Davis et.al [14] considered a multi-server queueing model in a priority queue and derived key inputs. Lot of literature [7,8,9,10,11] is available in this domain.
Customers arrive the queue in different ways based on the need of the service. Different arrival rates are also possible for the same kind of the service in real life. Studies are many in this area. An M/M/m queue with two classes of consumers and several vacations was examined by MingzhouXie et.al [12]. Studies [4,5,12,17] are significant in this domain.
In this paper, we present the transient analysis of M/M/C queue with two types of arrivals and state dependent service rates using R-K method. The work is presented as follows: Section 2describes the model, section 3 is about Transient state model and respective probabilities, Section 4, is of presentation of some constants of the system through numerical results and sensitivity analysis. Section 5 presents final conclusions.

II About the Model

We analyzed a finite Markovian Queue having multi -channel facility with heterogenous arrivals, state dependent service and Customer Impatience in Transient mode with the following assumptions:

1. The capacity of the system is considered as c(finite).
2. The number of service channels is c.
3. The mean arrival rate of Type-I customers is λ_1.
4. The mean arrival rate of Type-II customers is λ_2.
5. The mean service rate of servers of Type-I customers is μ_1.
6. The mean service rate of servers of Type-II customers is μ_2.
7. The number of phases is k.
8. The balking parameter is 1-b.
9. $\pi_{x,y}^{(t)}$ denotes the probability that there are “x” customers of Type-I and “y” customers of Type-II at time point “t”.

The following differential equations are formed to compute various probabilities:

$$\frac{d\pi_{0,0}^{(t)}}{dt} = -(\lambda_1+\lambda_2)b\pi_{0,0}^{(t)} + \mu_1 k\pi_{k,0}^{(t)} + \mu_2 k\pi_{0,k}^{(t)} \quad (1)$$

$$\frac{d\pi_{x,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1)\,\pi_{x,0}^{(t)} + \lambda_1 b\pi_{x-k,0}^{(t)} + (x+k)\mu_1\pi_{x+k,0}^{(t)} + \mu_2 k\pi_{x,k}^{(t)};\ k \le x \le (c-1)\mathrm{k} \quad (2)$$

$$\frac{d\pi_{0,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + y\mu_2)\,\pi_{0,y}^{(t)} + \lambda_2 b\pi_{0,y-k}^{(t)} + (y+1)\,\mu_2\pi_{0,y+k}^{(t)} + \mu_1 k\pi_{k,y}^{(t)};\ k \le y \le (c-1)\mathrm{k} \quad (3)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1 + y\mu_2)\,\pi_{x,y}^{(t)} + \lambda_1 b\pi_{x-k,y}^{(t)} + \lambda_2 b\pi_{x,y-k}^{(t)} + (x+k)\,\mu_1\pi_{x+k,y}^{(t)} + (y+k)\,\mu_2\pi_{x,y+k}^{(t)};\ x \text{ and } y \ne 0 \text{ and } x+y \le (c-1)\mathrm{k} \quad (4)$$

$$\frac{d\pi_{ck,0}^{(t)}}{dt} = -(ck\mu_1)\,\pi_{ck,0}^{(t)} + \lambda_1 b\pi_{(c-1)k,0}^{(t)} \quad (5)$$

$$\frac{d\pi_{0,ck}^{(t)}}{dt} = -(ck\mu_2)\,\pi_{0,ck}^{(t)} + \lambda_2\, b\pi_{0,(c-1)k}^{(t)} \quad (6)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -(x\mu_1 + y\mu_2)\pi_{x,y}^{(t)} + \lambda_1 b\pi_{x-k,y}^{(t)} + \lambda_2 b\pi_{x,y-k}^{(t)};\quad x \text{ and } y \ne 0 \text{ and } x+y = ck \quad (7)$$

III Performance Measures

Queueing measurements that are computed to forecast the system are

1. Expected length of system ($L_S^{(t)}$)
2. Mean waiting time ($W_S^{(t)}$)

IV Numerical Results

The numerical values for Mean length, waiting time and also the effect of various parameters are calculated by using MATLAB for Runge-Kuttafor the given values of the parameters. These values are taken by verifying the traffic intensity condition. They are presented in Tables 1-6 and Figures 1-6.

The model parameters are considered as follows:

$$c = 6, \lambda_1 = .01, \lambda_2 = .02, \mu_1 = .03, \mu_2 = .04, b = 0.001, k = 2$$

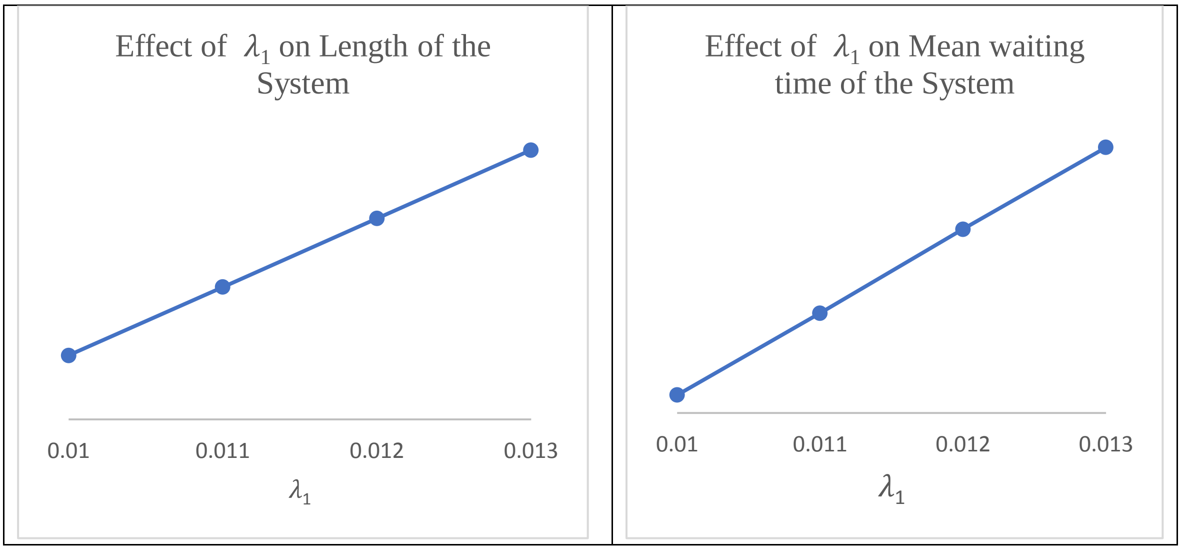
The time instances are taken as follows:

$$t_1 = 0.5, t_2 = 1.0, t_3 = 1.5, t_4 = 2.0$$

Table 1: Effect of λ_1

Parameter (λ_1)	t	t_1	t_2	t_3	t_4
0.01	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
0.011	$L_S^{(t)}$	0.0000108366	0.0000213530	0.0000315585	0.0000414625
	$W_S^{(t)}$	0.0009851541	0.0019412021	0.0028690030	0.0037693907
0.012	$L_S^{(t)}$	0.0000118218	0.0000232942	0.0000344275	0.0000452318
	$W_S^{(t)}$	0.0009851546	0.0019412040	0.0028690072	0.0037693978
0.013	$L_S^{(t)}$	0.0000128069	0.0000252354	0.0000372965	0.0000490011
	$W_S^{(t)}$	0.0009851551	0.0019412059	0.0028690113	0.0037694050

Figure 1: Effect of λ_1

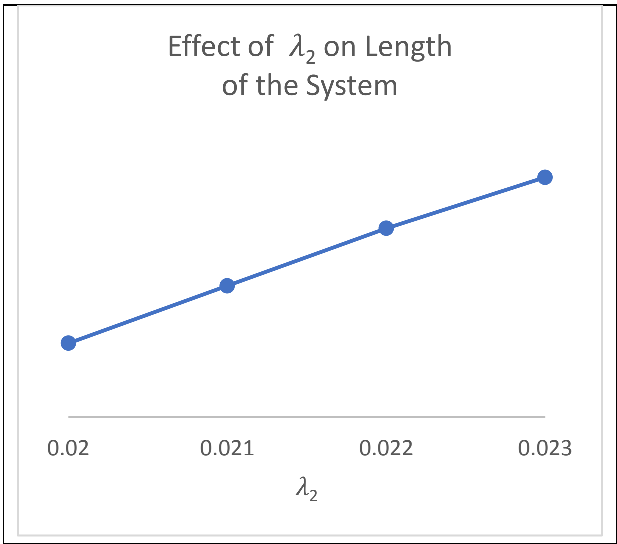


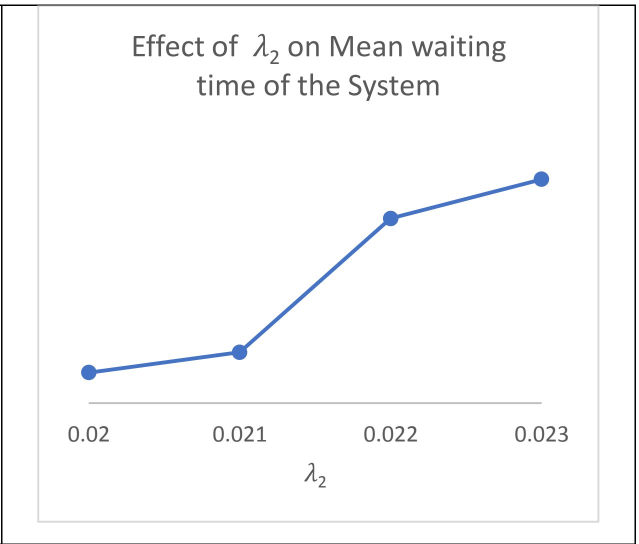
Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-I arrival rate λ_1.

Table 2: Effect of λ_2

Parameter (λ_2)	t	t_1	t_2	t_3	t_4
0.02	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
0.021	$L_S^{(t)}$	0.0000108366	0.0000213530	0.0000315585	0.0000414625
	$W_S^{(t)}$	0.0009746541	0.0019412020	0.0029148730	0.0038104107
0.022	$L_S^{(t)}$	0.0000109488	0.0000222942	0.0000344275	0.0000352426
	$W_S^{(t)}$	0.0009831246	0.0019568240	0.0032165072	0.0039672378
0.023	$L_S^{(t)}$	0.0000118069	0.0000243854	0.0000369765	0.0000478913
	$W_S^{(t)}$	0.0009851551	0.0020123059	0.0033049113	0.0040976050

Figure 2: Effect of λ_2





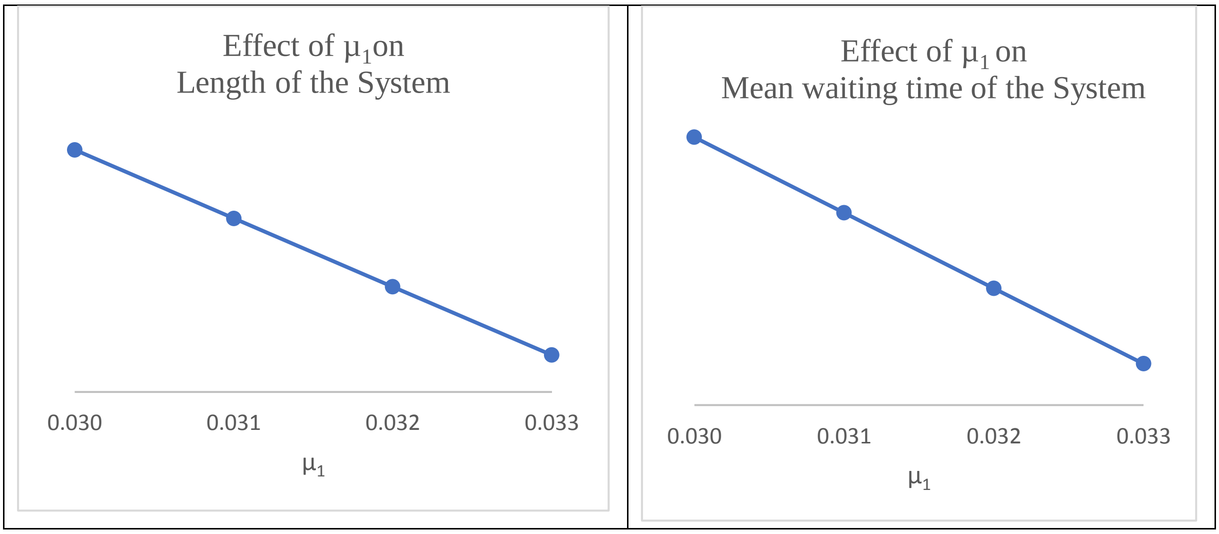
Inference: From the table and figures, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-II arrival rate λ_2.

Table 3: Effect ofμ_1

Parameter (μ_1)	t	t_1	t_2	t_3	t_4
0.03	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
0.031	$L_S^{(t)}$	0.0000098466	0.0000193926	0.0000286472	0.0000376194
	$W_S^{(t)}$	0.0009846637	0.0019392797	0.0028647641	0.0037620051

0.032	$L_S^{(t)}$	0.0000098417	0.0000193734	0.0000286050	0.0000375458
	$W_S^{(t)}$	0.0009841740	0.0019373617	0.0028605377	0.0037546462
0.033	$L_S^{(t)}$	0.0000098368	0.0000193543	0.0000285628	0.0000374724
	$W_S^{(t)}$	0.0009836847	0.0019354463	0.0028563197	0.0037473066

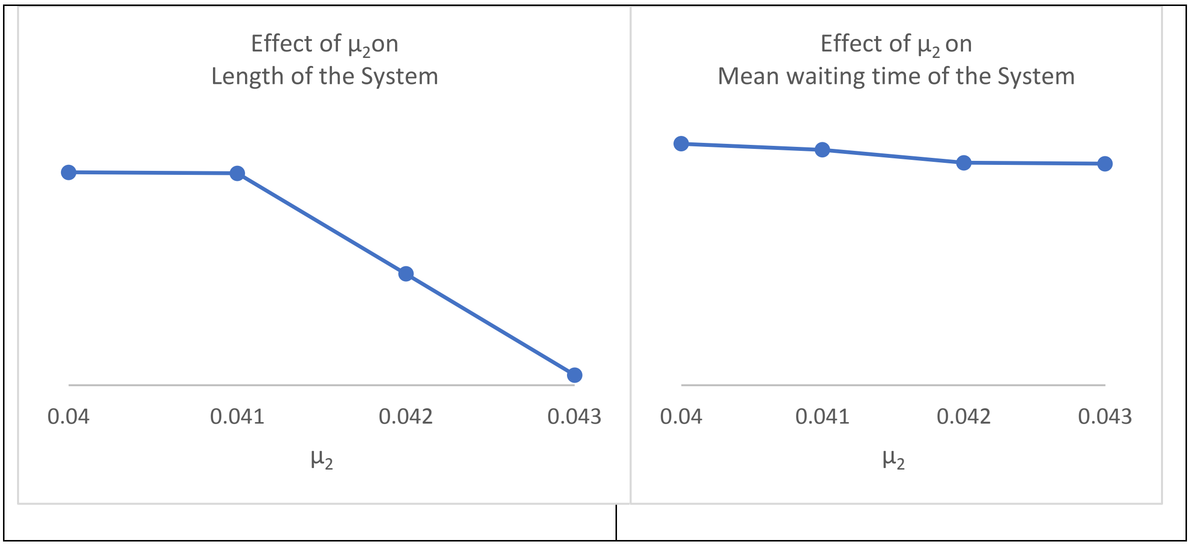
Figure 3: Effect of μ_1



Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_1.

Table 4: Effect of μ_2					
Parameter (μ_2)	t	t_1	t_2	t_3	t_4
0.04	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
0.041	$L_S^{(t)}$	0.0000098500	0.0000193628	0.0000286875	0.0000369981
	$W_S^{(t)}$	0.0009841436	0.0019368702	0.0027965988	0.0036947833
0.042	$L_S^{(t)}$	0.0000098477	0.0000192168	0.0000284563	0.0000369531
	$W_S^{(t)}$	0.0009796436	0.0018999702	0.0026429688	0.0035021831
0.043	$L_S^{(t)}$	0.0000098211	0.0000184118	0.0000282236	0.0000345631
	$W_S^{(t)}$	0.0009632536	0.0017603001	0.0026321487	0.0033654830

Figure 4: Effect of μ_2

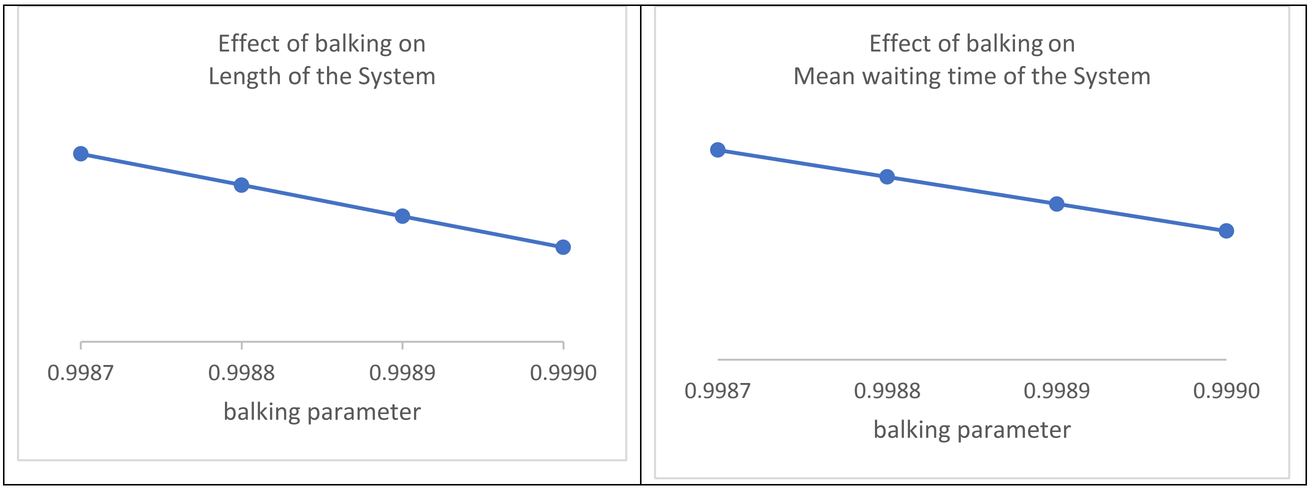


Inference: From the above table and figures, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_2.

Table 5 Effect of *balking parameter*

Parameter $(1-b)$	t	t_1	t_2	t_3	t_4
0.999	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
0.9989	$L_S^{(t)}$	0.0000108366	0.0000213530	0.0000315585	0.0000414625
	$W_S^{(t)}$	0.0010836695	0.0021353222	0.0031559030	0.0041463290
0.9988	$L_S^{(t)}$	0.0000118218	0.0000232942	0.0000344275	0.0000452318
	$W_S^{(t)}$	0.0011821855	0.0023294446	0.0034428079	0.0045232757
0.9987	$L_S^{(t)}$	0.0000128069	0.0000252354	0.0000372965	0.0000490011
	$W_S^{(t)}$	0.0012807016	0.0025235673	0.0037297136	0.0049002238

Figure 5:

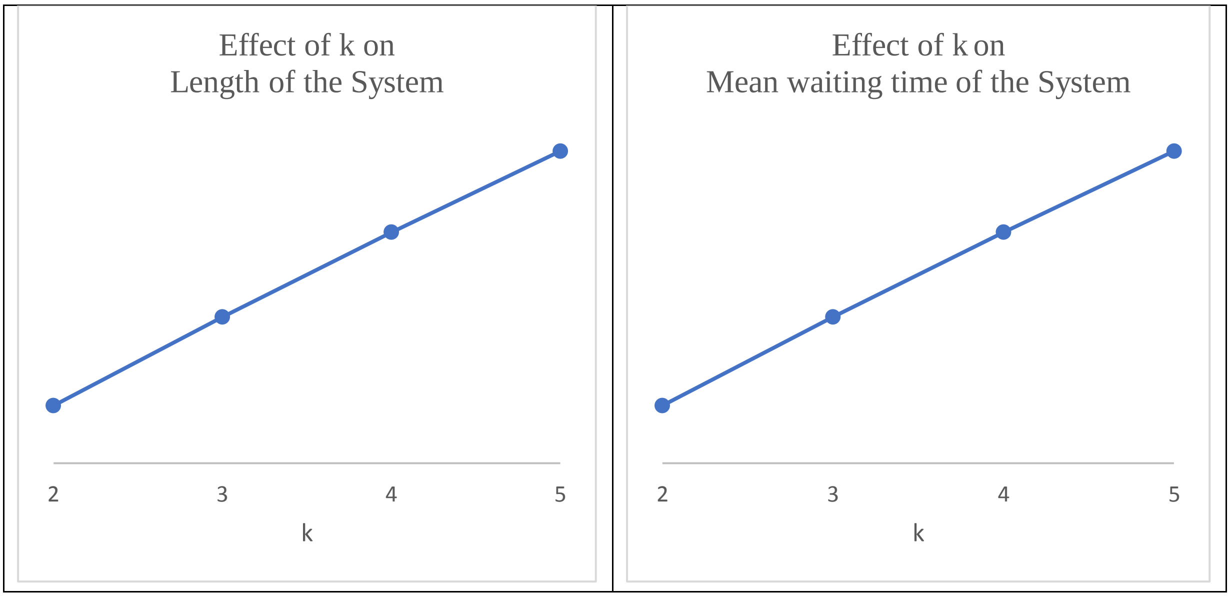


Inference: From the above table and figure, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to increase in balking probability.

Table 6: Effect of k

Parameter (k)	t	t_1	t_2	t_3	t_4
2	$L_S^{(t)}$	0.0000098515	0.0000194118	0.0000286896	0.0000376931
	$W_S^{(t)}$	0.0009851536	0.0019412002	0.0028689989	0.0037693835
3	$L_S^{(t)}$	0.0000146675	0.0000286896	0.0000420947	0.0000549098
	$W_S^{(t)}$	0.0014667575	0.0028689860	0.0042095226	0.0054910798
4	$L_S^{(t)}$	0.0000194118	0.0000376932	0.0000549099	0.0000711239
	$W_S^{(t)}$	0.0019411910	0.0037693508	0.0054910577	0.0071125075
5	$L_S^{(t)}$	0.0000240855	0.0000464306	0.0000671611	0.0000863937
	$W_S^{(t)}$	0.0024085608	0.0046431046	0.0067161993	0.0086395058

Figure 6:



Inference: From the table and figures, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in number of phases ”k”.

V Conclusion

Theobjective of this work was to detail a queueing system with two types of arrivals and state dependent service with customer’s impatience. We have also demonstrated sensitivity analysis of various parameters on performance measures of the system. This work can be extended by considering a cost function and to deduce the number of servers required to optimize such total cost.

References

1. A. M. Haghighi, J. Medhi and S. G. Mohanty, On a multi-server Markovian queueing system with balking and reneging. Computers Ops Res. (1986)13, 421-425.
2. Ancker C. J., Gafarian A., “Some queueing problems with balking and reneging”, Operations Research 11 (1963), pp. 88-100.
3. Artalejo J.R and V. Pla, On the impact of customer balking, impatience and retrials in telecommunication systems, Computers and Mathematics with Applications 57 (2009) 217-229.
4. BaraKim,JeongsimKim, OleBueker (2021), on-preemptive priority M/M/m queue with servers’ vacations, Computers & Industrial Engineering, Volume 160, October 2021, 107390.
5. E.P.C. Kao, S.D. Wilson, Analysis of non-pre-emptive priority queues with multiple servers and two priority classes, European Journal of Operational Research 118 (1999), 181-193.
6. F. A. Haight, Queueing with balking. Biometrika (1957),44, 360-369.
7. H.R. Gail, S.L. Hantler, B.A. Taylor, Analysis of a nonpreemptive priority multiserver queue, Advances in Applied Probability 20 (1988), 852-879.
8. J. F. Reynolds, The stationary solution of the multi-server queueing model with discouragement. Ops Res. (1968),16,6&71.
9. Kamlesh Kumar, Madhu Jain, and Chandra Shekhar (2019). Machine Repair System with F-Policy, Two Unreliable Servers, and Warm Standbys, Journal of Testing and Evaluation, Vol. 47, No. 1, pp. 361–383, https://doi.org/10.1520/JTE20160595. ISSN 0090-3973
10. Kuo-HsiungWang, Jyh-Bin Kea et.a(2005),Profit analysis of the M/M/R machine repair problem with balking, reneging, and standby switching failures, Computers & Operations Research 34 (2007) 835–847.
11. M. Harchol-Balter, T. Osogami, A. Scheller-Wolf, A. Wierman, Multi-server queueing systems with multiple priority classes, Queueing Systems 51 (2005), 331-360.
12. MingzhouXie, LiXia, JunXu, On M/G[b]/1/K queue with multiple state-dependent vacations: A real problem from media-based cache in hard disk drives, Performance Evaluation,(2020) Volume 139,1-20.
13. M. O. Abou-El-Ata, New approach for the moments of the simple birth-death processes and discrete distribution II. In Faculty of Education Journal (1987), pp. 53-62.

14. R.H. Davis, Waiting-time distribution of a multi-server, priority queuing system, Operations Research 14 (1966), 133-136.
15. R.O. Al-Seedy , A.A. El-Sherbiny, Transient solution of the M/M/c queue with balking and reneging, Computers and Mathematics with Applications 57 (2009) 1280 1285.
16. V.N. Rama Devi et.al, Analysis of a M/M/1 Queueing System with Two-Phase, N-Policy, Server Failure and Second Optional Batch Service with Customers impatient Behavior, IOP Conf. Series: Journal of Physics: Conf. Series 1344, 012015(2019).
17. V. P. Singh, Two server Markovian queues with balking: heterogeneous vs homogeneous servers. Ops Res.(1970). 18, 145-159.
18. Y. Levy, U. Yechiali, M/M/s queues with servers' vacations, INFOR 14 (1976), 153- 163.