Bipolar Valued Intuitionistic Multi Fuzzy Normal Subnear-Ring of a Near-Ring

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Abstract: In this paper, bipolar valued intuitionistic multi fuzzy normal subnear-ring of a near-ring is introduced and some theorems are stated and proved.

Keywords: Bipolar valued fuzzy subset, Bipolar valued multi fuzzy subset, Bipolar valued intuitionistic multi fuzzy subnear-ring, Bipolar valued intuitionistic multi fuzzy normal subnear-ring, Product and strongest bipolar valued intuitionistic multi fuzzy subnear-ring.

1. Introduction

In 1965, Zadeh [9] introduced the notion of a fuzzy subset of a universal set. Zhang [10, 11] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [3, 4]. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [4]. Anitha et al. [1, 2] introduced the bipolar valued fuzzy subgroup. Shyamala and Shanthi [7] have introduced the bipolar valued multi fuzzy subgroups of a group. Yasodara and Sathappan [8] defined the bipolar valued multi fuzzy subsemirings of a semi ring. Bipolar valued multi fuzzy subnear-ring of a near-ring has been introduced by Muthukumaran and Anandh [5]. In this paper, the concept of bipolar valued intuitionistic multi fuzzy normal subnear-ring of a near-ring is introduced and established some results.

2. Preliminaries

Definition 2.1 ([10]). A bipolar valued fuzzy set (BVFS) ℬ in X is defined as an object of the form ℬ = {〈x, ℬ⁺(u), ℬ⁻(u)〉/x ∈ X}, where ℬ⁺ : X → [0,1] and ℬ⁻ : X → [−1,0]. The positive membership degree ℬ⁺(u) denotes the satisfaction degree of an element x to the property
corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element $x$ to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

**Definition 2.2.** A bipolar valued Intuitionistic fuzzy set (BVIFS) $\mathcal{B}$ in $X$ is defined as an object of the form $\mathcal{B} = \{(x, \mathcal{B}^+(x), \mathcal{B}^-(x))/x \in X\}$, where $\mathcal{B}^+: X \rightarrow [0,1]$ and $\mathcal{B}^-: X \rightarrow [-1,0]$. The positive membership degree $\mathcal{B}^+(u)$ denotes the satisfaction degree of an element $x$ to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-(u)$ denotes the satisfaction degree of an element $x$ to some implicit counter-property corresponding to a bipolar valued fuzzy set.

**Definition 2.3.** A bipolar valued Intuitionistic multi fuzzy set (BVIMFS) $\mathbf{A}$ in $X$ is defined as an object of the form

$$\mathbf{A} = \left\{ \mathcal{B}^+_1(\alpha(x), \beta(x)), \mathcal{B}^-_1(\alpha(x), \beta(x)); \ldots; \mathcal{B}^+_n(\alpha(x), \beta(x)), \mathcal{B}^-_n(\alpha(x), \beta(x)) \right\} / x \in X,$$

where $\mathcal{B}^+_i: X \rightarrow [0,1]$ and $\mathcal{B}^-_i: X \rightarrow [-1,0]$ for all $i$. The positive membership degree $\mathcal{B}^+_i(u)$ denotes the satisfaction degree of an element $x$ to the property corresponding to a bipolar valued fuzzy set $\mathcal{B}$ and the negative membership degree $\mathcal{B}^-_i(u)$ denotes the satisfaction degree of an element $x$ to some implicit counter-property corresponding to a bipolar valued fuzzy set $\mathcal{B}$.

**Definition 2.4.** Let $\mathcal{R}$ be a near ring. A BVIMFS $\mathbf{A}$ is said to be a bipolar valued intuitionistic multi-fuzzy subnear-ring $\mathcal{R}$ (BVIMFNSNR) if the following conditions are satisfied, for all $i$

1. $\mathcal{B}^+_i(x - y) \geq \min\{\mathcal{B}^+_i(\alpha(x) - \alpha(y)), \mathcal{B}^+_i(\beta(x) - \beta(y))\}$
2. $\mathcal{B}^+_i(xy) \geq \min\{\mathcal{B}^+_i(\alpha(x), \beta(x)), \mathcal{B}^+_i(\alpha(y), \beta(y))\}$
3. $\mathcal{B}^-_i(x - y) \leq \max\{\mathcal{B}^-_i(\alpha(x) - \alpha(y)), \mathcal{B}^-_i(\beta(x) - \beta(y))\}$
4. $\mathcal{B}^-_i(xy) \geq \min\{\mathcal{B}^-_i(\alpha(x), \beta(x)), \mathcal{B}^-_i(\alpha(y), \beta(y))\}$, for all $x, y$ for all $x, y \in \mathcal{R}$

**Definition 2.5.** Let $\mathcal{R}$ be a near ring. A bipolar valued intuitionistic multi fuzzy subnear–ring of $\mathcal{R}$ is said to be a bipolar valued intuitionistic multi fuzzy normal subnear –ring (BVIMFNSNR) of $\mathcal{R}$ if

1. $\mathcal{A}^+_i(\alpha(x + y), \beta(x + y)) = \mathcal{A}^+_i(\alpha(y + x), \beta(y + x))$
2. $\mathcal{A}^-_i(\alpha(x + y), \beta(x + y)) = \mathcal{A}^-_i(\alpha(y + x), \beta(y + x))$
3. $\mathcal{A}^+_i(\alpha(xy), \beta(xy)) = \mathcal{A}^+_i(\alpha(xy), \beta(xy))$
4. $\mathcal{A}^-_i(\alpha(xy), \beta(xy)) = \mathcal{A}^-_i(\alpha(xy), \beta(xy))$ for all $x, y \in \mathcal{R}$ and for all $i$.

**Definition 2.6.** Let $\mathcal{A} = (\mathcal{A}^+_1, \mathcal{A}^+_2, \ldots, \mathcal{A}^+_n, \mathcal{A}^-_1, \mathcal{A}^-_2, \ldots, \mathcal{A}^-_n)$ be a bipolar valued intuitionistic multi fuzzy subset in a set $\mathcal{S}$, the strongest bipolar valued intuitionistic fuzzy relation on $\mathcal{S}$ that is bipolar valued intuitionistic multi fuzzy relation on $\mathcal{A}$ is

$$\mathcal{V} = \left\{ \left\{ \mathcal{V}^+_1(\alpha(x), \beta(xy)), \mathcal{V}^-_1(\alpha(x), \beta(xy)) \right\}, \ldots, \left\{ \mathcal{V}^+_n(\alpha(x), \beta(xy)), \mathcal{V}^-_n(\alpha(x), \beta(xy)) \right\} \right\} / x, y \in \mathcal{S}$$

Where $\mathcal{V}^+_i(\alpha(x), \beta(xy)) = \min\{\mathcal{A}^+_i(\alpha(xy), \beta(xy))\}$ and

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\[
\mathcal{V}_i^{-}(\alpha(xy), \beta(xy)) = \max\{\mathcal{A}_i^{-}(\alpha(xy), \beta(xy))\} \quad \text{for all } x, y \in \mathcal{S} \text{ and for all } i
\]

3. Properties

**Definition 3.1.** Let \( \mathcal{R} \) be a near ring and \( \mathcal{B} \) be an bipolar valued Intuitionistic multi fuzzy subset of a ring \( \mathcal{R} \). Then \( \mathcal{B} \) is called an bipolar valued intuitionistic multi fuzzy right ideal of \( \mathcal{R} \) if \( \mathcal{B} \) is an bipolar valued intuitionistic multi fuzzy sub near ring of \( \mathcal{R} \) and satisfies for all \( x, y \in \mathcal{R}, i = 1, 2, \ldots \):

i) \( \mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\} \)

\( \mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

ii) \( \mathcal{B}_i^+(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

\( \mathcal{B}_i^-(x - y) \geq \min\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

iii) \( \mathcal{B}_i^+((x + i)y - xy) \geq \mathcal{B}_i^-(i) \) & \( \mathcal{B}_i^+(y + x - y)I \leq \mathcal{B}_i^+(i) I \)

\( \mathcal{B}_i^-(x - y) \leq \mathcal{B}_i^-(\alpha(x, \beta(x)) \) & \( \mathcal{B}_i^-(y + x - y)I \geq \mathcal{B}_i^-(\alpha(y), \beta(y)) I \)

\( \mathcal{B}_i^-(y + x - y) \leq \mathcal{B}_i^-(\alpha(x, \beta(x)) \) & \( \mathcal{B}_i^-(y + x - y)I \geq \mathcal{B}_i^-(\alpha(y), \beta(y)) I \)

**Definition 3.2.** Let \( \mathcal{R} \) be a near ring and \( \mathcal{B} \) be an bipolar valued Intuitionistic multi fuzzy subset of a ring \( \mathcal{R} \). Then \( \mathcal{B} \) is called an bipolar valued intuitionistic multi fuzzy left ideal of \( \mathcal{R} \) if \( \mathcal{B} \) is an bipolar valued intuitionistic multi fuzzy sub near ring of \( \mathcal{R} \) and satisfies for all \( x, y \in \mathcal{R}, i = 1, 2, \ldots \):

i) \( \mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\} \)

\( \mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

ii) \( \mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\} \)

\( \mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

iii) \( \mathcal{B}_i^+(\alpha(xy), \beta(xy)) \geq \mathcal{B}_i^+(\beta(xy)) \) & \( \mathcal{B}_i^+(\alpha(xy)) \leq \mathcal{B}_i^+(\alpha(xy)) \)

\( \mathcal{B}_i^-(\alpha(xy), \beta(xy)) \leq \mathcal{B}_i^-(\beta(xy)) \) & \( \mathcal{B}_i^-(\alpha(xy)) \leq \mathcal{B}_i^-(\alpha(xy)) \)

iv) \( \mathcal{B}_i^+(y + x - y) \geq \mathcal{B}_i^+(\alpha(x, \beta(x)) \) & \( \mathcal{B}_i^+(y + x - y) \leq \mathcal{B}_i^+(\alpha(y), \beta(y)) \)

\( \mathcal{B}_i^-(y + x - y) \leq \mathcal{B}_i^-(\alpha(x, \beta(x)) \) & \( \mathcal{B}_i^-(y + x - y) \geq \mathcal{B}_i^-(\alpha(y), \beta(y)) \)

**Proposition 3.3.** If an bipolar valued intuitionistic multi fuzzy subsets of \( \mathcal{R} \) satisfies the properties

\( \mathcal{B}_i^+(x - y) \geq \min\{\mathcal{B}_i^+(\alpha(x - y), \beta(x - y))\} \), \( \mathcal{B}_i^-(x - y) \leq \max\{\mathcal{B}_i^-(\alpha(x - y), \beta(x - y))\} \)

then

i) \( \mathcal{A}_i^+(0) \geq \mathcal{A}_i^+(\alpha(x)) \) & \( \mathcal{A}_i^-(0) \leq \mathcal{A}_i^-(\alpha(x)) \)

ii) \( \mathcal{A}_i^+(-\alpha(x)) = \mathcal{A}_i^+(\alpha(x)) \) & \( \mathcal{A}_i^-(-\alpha(x)) = \mathcal{A}_i^-(\alpha(x)) \)

\( x, y \in \mathcal{R} \) & \( i = 1, 2, \ldots \)

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Proof
We have that for any $x \in R$, $i=1,2,\ldots$

i) \[ A^+_i(0) = A^+_i(\alpha(x) - \alpha(x)) \geq \min\{A^+_i(\alpha(x)), A^+_i(\alpha(x))\} = A^+_i(\alpha(x)) \]

Hence $A^+_i(0) \geq A^+_i(\alpha(x))$

$A^-_i(0) = A^-_i(\alpha(x) - \alpha(x)) \leq \max\{A^-_i(\alpha(x)), A^-_i(\alpha(x))\} = A^-_i(\alpha(x))$

Hence $A^-_i(0) \leq A^-_i(\alpha(x))$

ii) \[ A^+_i(-\alpha(x)) = A^+_i(0 - \alpha(x)) \geq \min\{A^+_i(0), A^+_i(\alpha(x))\} = A^+_i(\alpha(x)) \]

Hence $A^+_i(-\alpha(x)) = A^+_i(\alpha(x))$

$A^-_i(-\alpha(x)) = A^-_i(0 - \alpha(x)) \leq \max\{A^-_i(0), A^-_i(\alpha(x))\} = A^-_i(\alpha(x))$

Hence $A^-_i(-\alpha(x)) = A^-_i(\alpha(x))$

**Proposition 3.4.**

If $\mathcal{B} = (B^+_1, B^+_2, \ldots, B^+_n, B^-_1, \ldots, B^-_n)$ and $\mathcal{C} = (C^+_1, C^+_2, \ldots, C^+_n, C^-_1, C^-_2, \ldots, C^-_n)$ are two BVIMFNSNR with degree n of a near ring $R$, then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFNSNR of $N$.

Proof
Let $\mathcal{D} = \mathcal{B} \cap \mathcal{C}$, by two BVIMFSNRs with degree n of a near ring then their intersection $\mathcal{B} \cap \mathcal{C}$ is a BVIMFSNR of $N$. Let $\mathcal{D}$ be a BVIMFSNR of the near ring $N$.

Let $u, v \in N$, for all $i$

\[ D^+_i(u + v) = \min\{B^+_i(u(x) + v(x)), u(y) + v(y)\}, C^+_i(u(x) + v(x)), u(y) + v(y)\} \]

$= D^+_i(u + v)$ for every $u, v \in N$

\[ D^+_i(uv) = \min\{B^+_i(u(xy)), v(xy)\}, C^+_i(u(xy)), v(xy)\} \]

$= D^+_i(uv)$ for every $u, v \in N$

\[ D^-_i(u + v) = \min\{B^-_i(u(x) + v(x)), u(y) + v(y)\}, C^-_i(u(x) + v(x)), u(y) + v(y)\} \]

$= D^-_i(u + v)$ for every $u, v \in N$

\[ D^-_i(uv) = \min\{B^-_i(u(xy)), v(xy)\}, C^-_i(u(xy)), v(xy)\} \]

$= D^-_i(uv)$ for every $u, v \in N$
is a BVIMFNSNR of the near ring $N$.

4. Conclusion

Bipolar valued multi fuzzy normal subnear-ring of a near-ring is defined and their properties are proved. We can extend these concepts into many algebraic systems.

References


